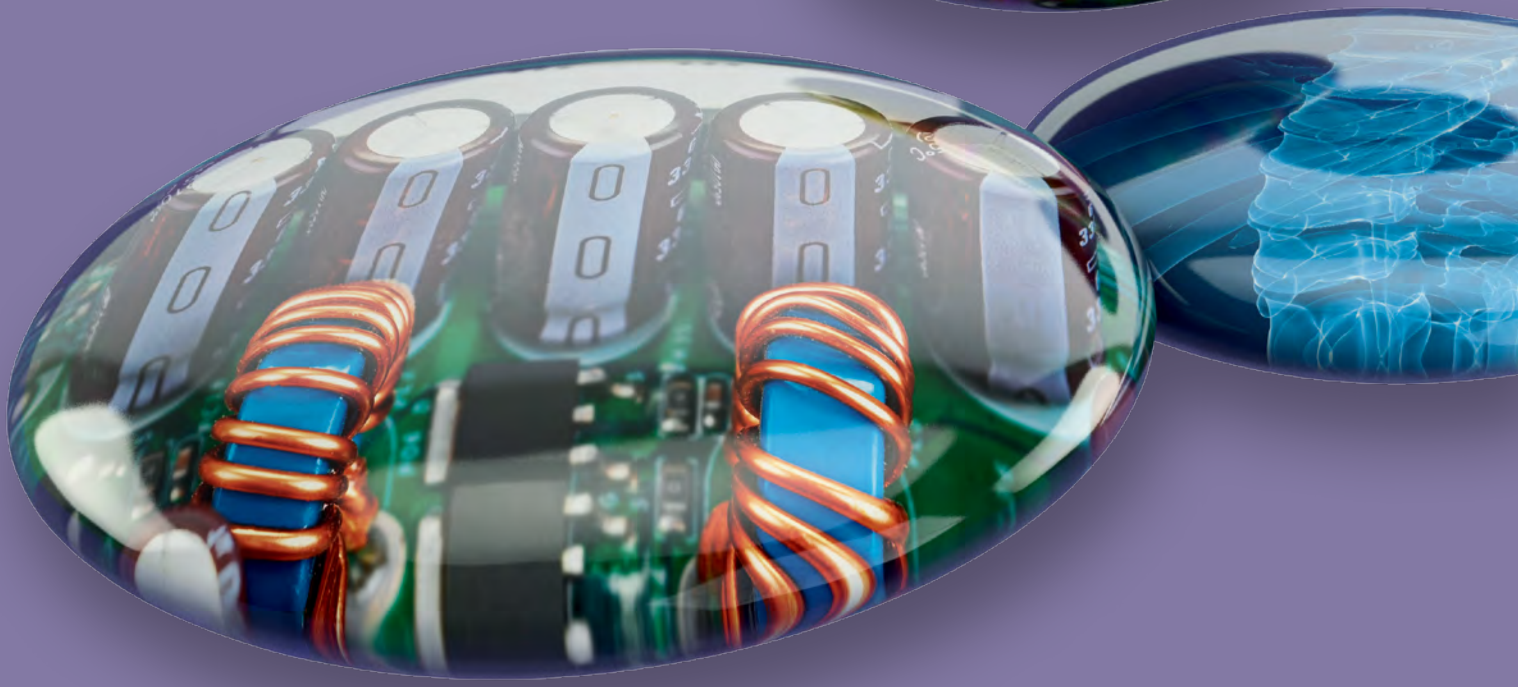
AS and A LEVEL

Mathematical Skills Handbook

PHYSICS A
PHYSICS B
(ADVANCING PHYSICS)

This Mathematical Skills Handbook is designed to accompany the OCR Advanced Subsidiary GCE and Advanced GCE specifications in Physics A and Physics B (Advancing Physics) for teaching from September 2015.



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Contents

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1 Introduction

In order to be able to develop their skills, knowledge and understanding in A Level Physics, learners need to have been taught, and to have acquired competence in, the appropriate areas of mathematics relevant to the subject as indicated in Appendix 5e of the specification.

The assessment of quantitative skills will include at least 40% Level 2 (or above) mathematical skills for physics (see below for a definition of 'Level 2' mathematics). These skills will be applied in the context of the relevant physics.

This Handbook is intended as a resource for teachers to clarify the nature of the mathematical skills required by the specification, and indicate how each skill is relevant to the subject content of the specification.

The content of this Handbook follows the structure of the table in section 5 of the specification, with each mathematical skill discussed in turn. The discussion of each skill begins with description and explanation of the mathematical concepts, followed by a demonstration of the key areas of the specification content in which the skill may be applied. Notes on common difficulties and misconceptions, as well as suggestions for teaching, may be included in either section as appropriate.

As this Handbook shows, all required mathematical skills can be covered along with the subject content in an integrated fashion. However, as assessment of the mathematical skills makes up a significant proportion of the overall assessment, OCR recommend that teachers aim to specifically assess learners' understanding and application of the mathematical concepts as a matter of course, in order to discover and address any difficulties that they may have. This is particularly relevant for learners who are not taking an A Level Mathematics qualification alongside A Level Physics.

Additional resources are available to support the teaching and learning of mathematical skills

http://www.ocr.org.uk/Images/179310-sketching-topic-exploration-pack-.pdf

http://www.ocr.org.uk/Images/179312-computation-topic-exploration-pack.pdf

http://www.ocr.org.uk/Images/179302-suvat-equations-topic-exploration-pack.pdf

Definition of Level 2 mathematics

Within A Level Physics, 40% of the marks available within written examinations will be for assessment of mathematical skills (in the context of physics) at a Level 2 standard, or higher. Lower level mathematical skills will still be assessed within examination papers, but will not count within the 40% weighting for physics.

The following will be counted as Level 2 (or higher) mathematics:

- application and understanding requiring choice of data or equation to be used
- problem solving involving use of mathematics from different areas of maths and decisions about direction to proceed
- questions involving use of A Level mathematical content (as of 2012) e.g. use of logarithmic equations.

The following will not be counted as Level 2 mathematics:

- simple substitution with little choice of equation or data and/or structured question formats using GCSE mathematics (based on 2012 GCSE mathematics content).

M0 – Arithmetic and numerical computation

M0.1 Recognise and make use of appropriate units in calculations

Mathematical concepts

Learners should be able to:

- give measurements and results of calculations in the correct units
- convert between different units
- determine the units for particular constants

Units indicate what a given quantity is measured in. A quantity without units is meaningless (with the exception of the few instances where quantities do not have units).

At GCSE learners will have used various units of measurement and would be required to recognise appropriate measures. For example whilst m is appropriate for a length/distance, learners should be able to identify that m^2 is used for area and m^3 is used for volume. Learners will be expected to use this skill at A Level.

Unit prefixes indicate particular multiples and fractions of units. A full list of SI unit prefixes is given in the table below, with the prefixes that are most likely to be used within the A Level Physics course highlighted.

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zeta	Z	10^{-2}	centi	c
10^{18}	Exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	µ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deca	da	10^{-24}	yocto	y

Learners would be expected to be able to convert between commonly encountered multiples without conversion ‘facts’ being given (e.g. 1 m = 1000 mm).

Converting between unit prefixes is a matter of multiplying appropriately. When converting quantity q from a factor 10^a to 10^b, the quantity needs to be multiplied by 10^{a-b}

For example converting 7 mm to µm requires a multiplication by $10^{-3--6} = 10^3$, thus 7 mm=7000 µm

In the A Level assessments, learners will be expected to be able to recognise and use compound units in the form m s^{-1}, rather than m/s. This is explained through the use of the power laws. Metres per second literally means we divide metres by time. This is written as

$$\frac{[\mathrm{m}]}{[\mathrm{s}]}$$

using square brackets to denote units rather than quantities. The reciprocal of s, 1/s can be written as s^{-1} and the units for velocity can be written as m s^{-1}.

Finding the units of a quantity involves extensive use of the power laws. The main forms of the power laws are:

$x^n \times x^m = x^{n+m}$ (multiplicative rule)

$\frac{x^n}{x^m} = x^{n-m}$ (division rule)

$(x^n)^m = x^{nm}$ (power rule)

$x^{-n} = \frac{1}{x^n}$ (reciprocal rule)

$x^{n/m} = \sqrt[m]{x^n}$ (root rule)

The *Data Formulae and Relationships booklet* details commonly used conversion factors for GCE Physics. Any other conversion to or from non-standard units that may be required in assessment will be provided in the question.

Within the A Level Physics course, learners will in general be expected to use and recognise standard SI units. For example, 10^{-3} m^3 is used rather than l (litre). The exception is the degree (°) for angles, which is used in addition to the radian. Kelvin and degree Celsius are both used for temperature.

Contexts in Physics

Learners will be expected to:

- Choose appropriate units of measure to use

When approximating gradients on graphs to measure rates of change learners will have to decide what the correct units of measurement are for the rate of change; {Amount of 'stuff' they are measuring}/{Unit of Time}.

- Be aware that the choice of units can have drastic changes on the output answer

A common misconception is that the learner will incorrectly say that if 1 cm is 10 mm then 100 cm^2 is 1000 mm^2. Actually 10 cm = 100 mm and hence the correct area in millimetres is 100 x 100 = 10 000 mm^2. The difference in these answers is a factor of 10 and can lead to massive calculation errors.

- Perform simple dimensional analysis to ensure the validity of a formula

For example when examining the quantity for kinetic energy, given by

$$E_{\mathrm{kinetic}} = \frac{1}{2}mv^2$$

learners should recognise that *E* has units kg m^2 s^{-2}, otherwise known as joule.

M0.2 Recognise and use expressions in decimal and standard form.

Mathematical concepts

Learners should be able to:

- Perform calculations in standard form
- Convert between standard and decimal form
- Use a calculator to express numbers in standard form
- Use common physical constants in standard form

The standard form expresses a number as $a \times 10^b$, with $0 < a < 10$ and b a +/- integer.

For example

$$F = 1.63 \times 10^3 \text{ N}$$

In physics many important constants are either very large or very small, the standard form is a convenient way of expressing these numbers. Using standard form also reduces calculation errors.

Learners are expected to express final results in standard from and to be able to convert to and from decimal form. The force above can be expressed as

$F = 16.3 \times 10^2$ N is *not acceptable*

$F = 0.163 \times 10^4$ N is *not acceptable*

$F = 1630$ N is *not acceptable*

$F = 1.63$ kN is *acceptable*

While the first three expressions above are numerically equivalent to the standard form above, these are not acceptable standard form. The final expression, using the prefix *kilo* is acceptable.

Calculator Use

Learners with access to a scientific calculator should be able to use it to convert between different decimal/standard form calculations. For Sharp calculators the 'Change' button is used whilst for Casio it is the "S➔D" button that performs this operation. Learners should be encouraged to use this calculator function to help with the accuracy of their answers.

To write numbers in standard form on a calculator either use a "$\times 10^x$" button (Casio) or the "EXP" button (Sharp).

For other models please encourage learners to investigate for themselves the appropriate functions.

Contexts in physics

- Any calculations which involve large or small numbers will require the use of standard form.

A lot of calculations in Physics involve large and small numbers. Formulae involving Newton's gravitational constant, the Boltzmann constant, the Planck constant, the speed of light, the fundamental electron charge and the masses of large /small bodies are typical examples of this. Common physical constants used are found in the data, formulae and relationships booklet and should be used to the accuracy stated unless otherwise specified.

For example in calculating the gravitational force between the Earth and the Sun, Newton's gravitational constant is required; 6.67 x 10^{-11} N m^2 kg^{-2}. The mass of the Earth is 5.79219 x 10^{24} kg, the mass of the Sun is 1.989x10^{30} kg and the *mean* distance between them is 1.496x10^{11} m. Applying the Newton's law of gravitation

$$F = \frac{GMm}{r^2}$$

gives:

$$F = \frac{6.67 \times 10^{-11} \times 5.79219 \times 10^{24} \times 1.989 \times 10^{30}}{(1.496 \times 10^{11})^2}$$

This can be worked out one step at a time using the power laws. Notice that the 1.496 also has to be squared

$$F = \frac{76.8428416197 \times 10^{43}}{2.238016 \times 10^{22}}$$

Finally using the division law:

$$F = 34.3352512313 \times 10^{21}\mathrm{N}$$

This can only be stated to 3 significant figures and needs to be put in standard form. Therefore

$$F = 34.3 \times 10^{21} = 3.43 \times 10^{22}\mathrm{N}$$

M0.3 Use ratios, fractions and percentages

Mathematical concepts

Learners should be able to:

- Calculate a percentage increase/decrease
- Calculate a percentage of a given amount
- Increase/decrease an amount by a percentage
- Understand the concept of ratio in calculations

There are many misconceptions when performing these calculations. This is often because learners are 'over-taught' the method on how to find percentage increases and decreases, actually the calculations are relatively easy and shouldn't provide them with too much fear. The key is to understand that the multiplier 1 represents a change of 0%. A multiplier of 1.43 therefore represents an increase of 43% whilst a multiplier of 0.83 represents a decrease of 17% (note that it is the difference between the multiplier and 1 which is the change – it isn't a percentage decrease of 83%). Amounts and percentages can then be found using a very simple formula

$$\text{Original Amount } \times \text{Multiplier} = \text{New Amount}$$

This formula can be stated in a formula triangle and then applied to situations where the percentage change is required. For example, initially the mass of a slowly evaporating liquid is 56 grams and after an hour it is 47 grams, to work out the percentage decrease we have:

$$56 \times \text{Multiplier} = 47$$

$$\text{Multiplier} = \frac{47}{56} = 0.84$$

This represents a percentage decrease of (1 − 0.84) = 16% decrease.

If we want to know the population of rabbits, of which initially there are 82 and subsequently experience an increase of 16% then:

$$82 \times \text{Multiplier} = \text{New Population}$$

$$82 \times 1.16 = \text{New} = 95$$

Contexts in Physics

Learners will be using these skills in finding:

- Efficiency calculations. They will need to use the formula

  $$\%\ \text{Efficiency} = \frac{\text{Output amount}}{\text{Input amount}} \times 100\%$$

  to find the percentage efficiency. This can be used in power calculations and calculations involving circuits.
- Use scales for measuring or representation on a graph. This can be denoted by a sentence, (i.e. 1 cm represents 10 m) or a ratio (1:1000). Maps are familiar examples learners will have encountered

M0.4 Estimate Results

Mathematical concepts

Learners should be able to:

- Make estimates of quantities by comparing to 'known measures'
- Make estimations in calculations

Being able to make an estimate for the size of a given measure is a notoriously difficult concept. The best advice for making reasonable estimates is to start at a 'known' quantity and then extrapolate from that fact to the object to be estimated. For example, let's say the mass of a tiger is to be estimated. If the mass of an average human is 'known' to be 70 kg then perhaps a sensible first estimate is to say that the tiger is 2 times the mass of the human. Hence the mass of the tiger could be estimated as 70 x 2 = 140 kg.

Estimation is also a valuable tool to check calculations for accuracy. For example the calculation:

$$4900/5.1$$

could be estimated as 5000/5 = 1000. This quick check can validate a learners calculated answer of 960.

In investigating the effects different parameters have on outputs on a particular problem then a good knowledge of the rules of mathematics is required. For example take the fictional formula

output

$$A = \frac{\overbrace{XY}^{\text{numerator}}}{\underbrace{Z \times a^{t}}_{\text{denominator}}}$$

exponent

When predicting how the value of *A* changes as a particular variable is increased or decreased, the following rules are useful

- The larger a denominator becomes the smaller the output is and vice versa
- The larger a numerator becomes the larger the output
- When t increases
  - a^t tends to infinity for a>1
  - a^t tends to zero for a<1
  - $a^t = 1$ for a=1
  - when t=0 then $a^t = 1$

In the example above assume that *X*, *Y* and *Z* are kept constant and *a*>1. *t* now increases from 0 to something large. As a^t increases the denominator will increase. As the denominator gets larger the overall fraction will get smaller and hence *A* will decrease, eventually tending asymptotically to 0.

Contexts in Physics

Opportunities for estimating quantities in Physics are numerous. A learner may want to estimate the time period of a waveform or estimate a typical value for the density of a substance.

Learners need to be able to estimate the effect of changing parameters in equations. An example is estimating the effect of increasing the force on the acceleration of an object.

In problem solving questions, a sensible approach to estimating quantities and the effects of changes is important. Where a calculation or experiment is performed an estimate can be performed to determine if the answer is sensible.

For example, when determining the density of an object it is useful to remember that water has a density of 1×10^3 kg m^{-3}. The densest metals have a density slightly above 2$\times$ 10^4 kg m^{-3} while Styrofoam has a density of around 75 kg m^{-3}. Air has a density of 1.2 kg m^{-3}.

If the result of a calculation or experiment shows that a liquid has a density of 10 kg m^{-3}, a quick comparison to the density of water shows that a calculation or measurement error is likely.

M0.5 Use a calculator to use power, exponential and logarithm functions

Mathematical concepts

Learners should be able to:

- Use a calculator to perform calculations involving powers of numbers, exponentials and logarithms (with different bases)

Multiplications such as 4 × 4 × 4 can be written in power form as 4^3. The chief difficulty with calculating powers on a calculator is the potential wide range of models that learners will own which all have different ways of entering powers and using them. For example different calculators will have “ x^y ”, “$■^■$" or “^” as the symbols for the power function. Learners should be encouraged to find the appropriate button on their own model and learn how to use it effectively.

The exponential function has the common form e^x, where e is a mathematical constant (approximately 2.718). It is sometimes written as exp(*x*) and models relationships in which the rate of change of a quantity is dependent on the instantaneous amount of the quantity. On a calculator it appears as e^x or $\mathrm{e}^{\blacksquare}$.

The chief difficulty for using logarithms is that in different contexts different bases need to be used. Learners need to be aware that a log to base e is denoted by ln for some calculators and that in general the "log" button on a calculator will mean log to base 10.

Contexts in physics

Learners should be expected to use this in:

- Using formulae involving exponentials such as radioactivity or any physical situation where growth and decay are used.

Example

The equation

$$N = N_0 \mathrm{e}^{-\lambda t}$$

represents an exponential decay with initial 'amount' N_0.

Let's say that the initial amount is 100 and $\lambda = 0.1$ so

$$N = 100\mathrm{e}^{-0.1t}$$

To find the amount at time t is relatively straightforward.

For example at $t = 2$

$$N = 100\mathrm{e}^{-0.1\times 2} = 100 \times \mathrm{e}^{-0.2} = 81.87$$

The 'opposite problem' is more challenging: given a value of N, can we find t?

For example, find at what time $N = 50$. Substituting $N = 50$ into the equation gives

$$50 = 100\mathrm{e}^{-0.1t}$$

Step 1: Divide by coefficient of exponential function

$$50/100 = 0.5 = \mathrm{e}^{-0.1t}$$

Step 2: Take natural logarithms, this is the inverse operation of the exponential

$$\ln(0.5) = -0.1t$$

Step 3: Divide by coefficient of *t*

$$\ln 0.5/-0.1 = t$$

$$t = 6.93$$

M0.6 Use calculators to handle sin x, cos x and tan x when x is expressed in radians or degrees

Mathematical concepts

Learners should be able to:

- Use calculators to handle sin *x*, cos *x* and tan *x* when *x* is expressed in radians and degrees

[Note: The specific examples of when the trigonometry functions are used and how to use them is covered in M4, this section just covers calculator use]

Learners need to ensure that:

- The calculator is in the correct mode

This requires learners to investigate their own calculator and find the function that changes from radians to degrees and vice versa. Encourage learners to check the display of their calculator before any calculation to ensure that they are in the mode they require. Some models have DEG/RAD/GRAD on the screen whilst others just have D/R/G.

- The inverse sin/cos/tan functions are used correctly

Often this requires learners to press the 2nd Function button before pressing the sin/cos/tan button. Make sure learners know how to do this. Additionally emphasise that for $\sin^{-1}$ and $\cos^{-1}$ this function will only be valid between -1 and 1. They should interpret a MATH ERROR on their calculator as a physical impossibility and not just accept it as an answer. For $\tan^{-1}$, *x* can be any value. Please note that some A level maths syllabi use the alternative notation arcsin, arccos and arctan for the inverse functions. This notation has the advantage in that learners do not confuse $\sin^{-1}$ as 1/sin *x* which is a common, yet obvious error. For example if we calculate $\sin^{-1}(1/2)$ the following buttons should be pressed:

$$\boxed{2^{nd}\ Func}\ \boxed{sin}\boxed{(}\boxed{1}\ \boxed{\div}\ \boxed{2}\boxed{)} = 30$$

Note: Some calculators require the number to be entered first, before pressing the function button.

Contexts in physics

Please see section M4 for a detailed explanation of the use of trigonometry in physics.

M1 – Handling Data

M1.1 Use an appropriate amount of significant figures

Mathematical concepts

Learners should be able to:

- Be able to round to a given number of significant figures or decimal places
- Give a result of a calculation to a level of accuracy appropriate to the level of the inputs

Learners must understand that the lowest level of significant figures in the inputs of a calculation will determine the number of significant figures in the answer. If there are 3 inputs to a particular calculation and they are quoted as being correct to respectively 2, 3 and 4 significant figures then the answer can only be quoted to 2 significant figures.

Learners will be familiar on rounding numbers in other contexts and in itself this should pose few problems. It is important to recognise that a zero at the end of a number is considered a significant figure.

For example, rounding 4.99 x 10^5 to 2 significant figures would yield 5.0 x 10^5. The 0 in this context represents a significant figure, not a place holder, and should be retained. Omitting the zero and quoting the answer as 5 x 10^5 is incorrect, as the result now only has one significant figure.

Contexts in Physics

Learners will be expected to:

- Report calculations to an appropriate number of significant figures given raw data quoted to a varying number of significant figures.
- Understand that calculated results can only be reported to the limits of the least accurate measurement.

Example

The electron mass is given to 1sf as 9×10^{-31} and the speed of light is given to 3sf as 3.00×10^8. In calculating the rest energy using $E = mc^2$, the answer can only be given correctly to 1 significant figure as this is the least accurate measurement;

$E = 9 \times 10^{-31} \times 9.00 \times 10^{16} = 81 \times 10^{-15} = 8.1 \times 10^{-14} = 8 \times 10^{-14}$.

Learners also need to be aware that when making measurements, these issues manifest themselves through the accuracy of their measuring tools. More information on this can be found in the practical skills handbook.

M1.2 Find arithmetic means

Learners should be able to:

- Find the arithmetic mean of a set of data in a list and in a table
- Understand the role outliers can have in a mean calculation and treat them accordingly

The mean is calculated using a simple formula:

$$\bar{x} = \frac{\sum x}{n}$$

where *x* represents a number data values and *n* is the number of data values. Most learners will be familiar with calculating the mean from GCSE and it is actually best taught as a rather ad-hoc message; 'Add them all up, divide by how many'.

Outliers (items of data significantly different from the mean) can also pose problems. It should be emphasised that there is no hard and fast rule on how to deal with outliers, they should be treated case by case. The detail involved to identify outliers goes beyond the scope of A level Physics, but for experiments a simple checklist is this:

- Was the suspected outlier recorded in error?
- Was the suspected outlier recorded in different conditions to the rest?

If the answer to any of these questions is YES then the outlier should be recorded with a reasoning for its rejection. The value is omitted from the data set in further analysis and the average should be calculated without this member. If a potential outlier is spotted at the time of the experiment then learners should repeat the measurement whenever practical to do so.

Calculator Use

Many scientific calculators have a Statistics mode where the mean can be calculated automatically. Whilst these functions are useful to gain full statistics (standard deviation, sum of squares, correlation coefficients etc.) the computational advantage of doing this for just the mean is nil. Learners can use this but will have to press just as many buttons and therefore have the same risk for error as a learner who decides to add these up on a calculator.

Contexts in physics

- Calculate a mean for repeated experiment readings

For example, imagine a group of learners are measuring the time t a paper cup takes to fall to the bottom of a large stairwell to investigate the effect of air-resistance. To improve the accuracy the experiment is repeated three times. Three measurements give

$$t = 3.42, 3.51, 3.70$$

The arithmetic mean is then

$$\bar{t} = \frac{3.42, 3.51, 3.70}{3} = 3.54$$

When calculating the mean it is acceptable but not necessary to add one significant figure and $\bar{t} = 3.543$ would be accepted. Adding more than one significant figure or reducing the number of significant figures is unacceptable.

When asked to comment on the absolute uncertainty in a combined measurement, learners are expected to use half the range.

M1.3 Understand simple probability

Mathematical concepts

Learners should be able to:

- Understand probability is measured on a scale from 0 to 1, 0 being impossible and 1 being inevitable
- Understand what a 'random process' is and appreciate that results can vary due to this

The fundamental misconception when dealing with probability is the idea of 'randomness'. Unfortunately in the last couple of years the word 'random' has entered the vernacular as a term for something bizarre happening. Learners could confuse a random event as being a 'rare' event rather than the actual meaning. There are a number of different definitions for the word 'random' but to keep it simple emphasise to learners that a random process doesn't produce rare results but it produces results that are *impossible to predict*.

Randomness is based on our inability to predict what will happen at a *given* moment of time. In nuclear physics for example take an isotope which has a long half-life and therefore a very low probability of the nucleus decaying and emitting a particle. The probability exists, but it is impossible for us to guess exactly when the nucleus will decay; this is a random process. We can make predictions on a global scale about a large number of nuclei, i.e. we can use the half life to say that although we cannot predict when a particular nucleus will decay, we *can* say that half of the nuclei will have decayed when a half life has elapsed.

Using a more mathematical model, imagine we rolled a die 1200 times. At a given moment it is impossible to predict the outcome of the next roll of the die but we can say that we can expect that after the whole 1200 rolls around 200 will be sixes. Probability helps us make predictions on the global scale not the local scale where randomness prevents us from making predictions. A key analogy is that any process that could be *feasibly* modelled by a throw of a die is a random event.

Contexts in Physics

- Understand probability in the context of decay

Example

Radioactivity is process that is continuous in time. But it is possible to discretise the process using a die. Imagine a roll of a die represents the random process of a nucleus decaying in a given time period, say for sake of simplicity the probability is 1/6. Imagine that the particle has initially 1200 particles. The half-life measures how long it would take for half of the nuclei to decay, for this example 600. The probability of 1/6 means that on average 1 'time period' out of every 6 will have a decaying nucleus. This means that for 600 nuclei to decay then there would have to be on average 600 × 6=3600 rolls or 'time periods'. Therefore the half-life in this discrete example is 3600 rolls. For a particle of mass N_0 initially and with a probability of p then the half-life is given by $N_0/2p$. This is analogous to the continuous model using:

$$N = N_0 e^{-\lambda t}$$

where N_0 is the initial mass, λ is a constant and t is the time. To find the half life $N = N_0/2$. Substituting this in:

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

Solving for t by initially cancelling the N_0 on both sides:

$$\frac{1}{2} = e^{-\lambda t}$$

Taking natural logs:

$$\ln\left(\frac{1}{2}\right) = -\lambda t$$

Using the log law; log(1/*x*)=−log *x* and then cancelling the negatives and dividing by λ:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

which is analogous to the discrete version. λ is called the decay constant.

M1.4 Make order of magnitude calculations

Mathematical concepts

Learners should be able to:

- Perform calculations using variables of different magnitudes in order to make a judgement on the final order of magnitude.

Order of magnitude calculations can be used to approximate values without extensive calculations and are also a useful check to ensure calculated values are reasonable.

Contexts in Physics

An architect has been commissioned to prepare a design for a large square of 150 by 150 m. His design includes a circular feature made with cobblestones covering most of the square. In the middle of the circle, there is a large fountain of 20 m diameter. Dotted around the square are trees and benches. The cobblestones are nearly square but vary in length between 10 and 30 cm with an even distribution of sizes. Give an order of magnitude estimate of the number of stones that are used on the square?

At first, the information may appear somewhat imprecise, but a reasonable estimate can be made despite this. Essentially the two pieces of information needed are the area covered by the cobblestones *A* and the average area covered by a single cobblestone *a*.

To find the area *A* we can use the area of a circle

$$A = \frac{\pi}{4} d^2$$

We thus need to find *d*. A reasonable estimate for the outside diameter is 150 m, as the circle covers nearly the entire square. By looking at the equation, we see that the diameter is squared. While the fountain is large, the area taken up by the fountain is very small compared to the area of the entire square. Similarly, we can ignore the area taken up by the trees and other features.

The average area *a* a cobblestone covers poses more difficulty. While it is tempting to take the average length as 20 cm, we are interested in the average area. This can be found by

$$a = \frac{30^2 + 10^2}{2} = 500\ \mathrm{cm}^2$$

We now have both *a* and *A*, which gives

$$n = \frac{A}{a} = \frac{\pi}{4} 150^2/0.05 = 3.5 \times 10^5$$

Rounding up to 1 significant figure gives an estimate of $n = 4 \times 10^5$stones.

M1.5 Identify uncertainties in measurements and use simple techniques to determine uncertainty when data are combined by addition, subtraction, multiplication, division and raising to powers

Mathematical concepts

Learners should be able to:

- appreciate that uncertainties exist when taking measurements
- determine absolute and percentage uncertainties
- determine the uncertainty in the final value when adding or subtracting readings.

Mathematical concepts

When a measurement is taken there will be an uncertainty due to the level of precision of the measuring apparatus. When multiple measurements are combined, the uncertainty in the final result will be a combination from the individual uncertainties in each measurement.

When adding or subtracting measurements, the absolute uncertainties are simply added together

combined uncertainty = uncertainty in A + uncertainty in B

The *absolute* uncertainty is the amount a measurement could be 'out' by. The *percentage* uncertainty is the ratio of the absolute uncertainty to the quantity measured.

$$\%\ \text{uncertainty} = \frac{\text{absolute uncertainty}}{\text{quantity measured}} \times 100\%$$

When multiplying or dividing, the percentage uncertainties need to be added together.

Raising to a power is a special case of a multiplication, in this case the percentage uncertainty is multiplied by the power the value is being raised to.

Contexts in physics

A ruler is graduated in divisions every 1 mm. A ruler is a non-digital device, we record to the figures on the scale plus one that is estimated.

Using the half-division rule, the estimation is 0.5 mm. The overall uncertainty in any distance measured always comes from two readings, so the overall uncertainty = 2 × 0.5 mm = 1 mm.

In a distance measurement covering the entire 300 mm length of the ruler, the uncertainty is small

$$\text{percentage uncertainty} = \frac{2 \times 0.5}{300} \times 100\% = 0.3\%$$

For shorter distances, the percentage uncertainty becomes more significant. For measuring a distance of 25 mm:

$$\text{percentage uncertainty} = \frac{2 \times 0.5}{25} \times 100\% = 4\%$$

Example

The difference in length of a rod due to a change in temperature is to be found. The absolute uncertainties of both measurements are summed up to give the uncertainty in the change in length.

An experimenter uses a rule to determine the elongation of a metal rod due to thermal expansion, and records measurements and uncertainties in the table below

Quantity	Value	Absolute Uncertainty	Percentage Uncertainty
Length when cold	54.3 cm	0.1 cm	0.1/54.3 × 100% = 0.18%
Length when hot	55.2 cm	0.1 cm	0.1/55.2 × 100% = 0.18%
Increase in length	0.9 cm	0.2 cm	0.2/0.9 × 100% = 22%

While there is a negligible percentage uncertainty in each length measurement, the percentage uncertainty in the elongation is much greater and care should be taken to ensure the measurement technique and apparatus are appropriate.

To determine the power dissipated in an electronic component the potential difference V and the current I are measured. The percentage uncertainties are combined in the table below to find the uncertainty in the value for P.

Quantity	Value	Absolute Uncertainty	Percentage Uncertainty
V	5.0 V	0.1 V	0.1/5.0 × 100% = 2%
I	1.3 A	0.1 A	0.1/1.3 × 100% = 7.7%
P	6.5 W	9.7% of 6.5 = 0.63 W	2% + 7.7% = 9.7%

M2 – Algebra

M2.1 Understand and use the symbols $= < \ll > \gg \propto \approx \ \Delta$

Mathematical concepts

Learners should be able to:

- Use these symbols appropriately and correctly in their given contexts
- Understand these symbols in the contexts of formulae given

Learners should have had exposure to the symbols $= < \ll > \gg \approx$ from an early age, and should understand how and why they are used.

The symbol $\propto$ means 'is proportional to'. If two quantities *A* and *B* are *directly proportional* then the appropriate mathematical statement is

$$A \propto B$$

If the two quantities are *inversely proportional* then the appropriate relationship is:

$$A \propto \frac{1}{B}$$

The symbol $\sim$ means 'is roughly equal to' or 'of the same order'. This symbol may be used in the context of approximations made in calculations of quantities, to indicate that a formula in which an approximation has been made is 'roughly equal to' the original formula.

The Δ symbol represents a change in a quantity from its original state. For example

$$\Delta p = p_{final} - p_{start}$$

The change in quantity can be both positive and negative.

Note that the Δ symbol has another meaning in physics, the uncertainty in a measurement.

Contexts in Physics

Learners will be expected to:

- Recognise the significance of the symbols in the expression $F \propto \frac{\Delta v}{\Delta t}$

Newton's second law formally states that the force is proportional to the rate of change of velocity. It is important for the learner to know that a *rate of change* represents (in most cases) a change over *time*. Therefore they should understand that the rate of change of velocity means $\frac{\Delta v}{\Delta t}$.

While not explicitly stated, the force is directly proportional to the rate of change of velocity

$$F \propto \frac{\Delta v}{\Delta t}$$

To proceed from here the proportionality symbol is replaced by an equals sign and a *constant of proportionality k* is introduced:

$$F = k\frac{\Delta v}{\Delta t}$$

In this case, *k* represents the mass *m* of an object.

This concept is also applied in the study of radioactivity. The rate of *decrease* of the number of nuclei is proportional to the instantaneous number of nuclei from a certain unstable isotope. The word 'decrease' is key here as it necessitates the use of a minus sign in the statement and hence

$$\frac{\Delta N}{\Delta t} \propto -N$$

Another example is Newton's law of gravitation where the force between two objects is inversely proportional to the square of the distance between them and the following statement is formed

$$F \propto \frac{1}{r^2}$$

From here the proportionality symbol is replaced by a constant k, and an equals sign as before

$$F = k\frac{1}{r^2}$$

here *k* is the product of the mass of the two objects and the gravitational constant.

M2.2 Change the subject of the equation, including non-linear equations

Learners should be able to:

- Rearrange a linear formula to change the subject
- Rearrange a non-linear formula to change the subject

Linear equations occur frequently in physics. An equation is linear if all variables in the equation only have powers of 1 and it does not contain a product or fraction of different variables. The general form of a linear equation is

$$y = ax + b$$

With *a* and *b* constants.

The equations below

$$z = x^2$$

$$z = \frac{1}{x}$$

$$z = \sqrt{x}$$

$$z = xy$$

$$z = \frac{x}{y}$$

$$z = \sin(x)$$

$$z = \cos(x)$$

$$z = \log(x)$$

$$z = e^x$$

are all non-linear when *x* and *y* are both variables .

When applying the above rules to determine if an equation is linear, care should be taken to distinguish between a variable and a constant. For example the equation

$$s = vt$$

is linear if v can be considered a constant. If v is a variable then the equation is nonlinear. The simplest way to see this is to compare an object travelling at a constant speed $v = 5$ ms^{-1} to an object accelerating such that $v = 2t$. After substituting the velocity into the equation above

$$s = 5t \text{ (constant velocity)}$$

$$s = 2t^2 \text{ (accelerating)}$$

it is clear that in the first case the result will be a linear graph and in the second a parabola.

In general non-linear equations are more difficult to master but in the context of A Level Physics they shouldn't pose any more difficulties than linear ones.

Care should be taken as to the order the operations as the correct sequence depends on the equation. A good strategy is to tell the learners that the variable that is to be made the subject has to put 'on its own' on one side of the equation. The variable may be 'locked' up in a square or root and they must first do the inverse of these operations to 'unlock' the variable and therefore change the subject. See below for some worked examples.

Contexts in physics

- Changing the subject in a suvat equation

Example

The suvat equation for velocity is a linear equation:

$$v = u + at$$

To change the subject to *t*, it should be noted that first *u* on the RHS has to be 'unlocked'. The opposite of adding u is subtracting it (remembering it has to be done on both sides)

$$v - u = at$$

The final part is to see that t is still 'locked' by *a*. Therefore the opposite is division by *a* and hence:

$$t = \frac{v - u}{a}$$

and the subject has been changed.

- Changing the subject in a non-linear equation like $E = mc^2$

Example

Rearranging this for *c* means first of all that we have to divide by *m*. Notice the square isn't applied to *m*, otherwise it would have been written as $E = (mc)^2$ or m^2c^2. Therefore

$$\frac{E}{m} = c^2$$

finally we have to take the square root of both sides

$$c = \sqrt{\frac{E}{m}}$$

we now have *c* on its own on the LHS.

M2.3 Substitute numerical values into algebraic equations using appropriate units for physical quantities

Mathematical concepts

Learners should be able to

- Substitute values into an expression to calculate a quantity from a formula

Learners should be aware of the principles from GCSE Maths and this should pose few difficulties. The most common problem is dealing with powers and negative quantities in formulae. The expression x^2 can cause issues when a negative number is substituted. Substituting $x = -2$ should be calculated as $(-2)^2 = 4$ **not** $-2^2 = -4$. These problems can come to the fore in formulae where powers occur such as the inverse square law, kinetic energy and suvat equations.

Additionally in the context of a formula such as $p = mv$ learners should be aware that the mass is being *multiplied* by the velocity despite the absence of a multiplication sign. In general the laws of BIDMAS should be adhered to where the operations should be completed in the order of Brackets, Indices, Division, Multiplication, Addition and Subtraction.

Contexts in Physics

- Substituting values into expressions to find quantities such as momentum
- Substituting values into expressions involving powers and fractions

Example

The formula for linear momentum is given by $p = mv$ where *m* is the mass and *v* is the velocity. If the mass of an object is 2 kg and the velocity is 4 m s^{-1} then the linear momentum is calculated as:

$$p = mv$$

$$p = 2 \times 4 = 8 \text{ kg ms}^{-1}$$

Example

A suvat equation after rearrangement can be written as

$$a = \frac{v^2 - u^2}{2s}$$

After substituting *v* = 5, *u* = -4 and *s* = 2 the value for *a* can be calculated

$$a = \frac{(5^2 - (-4)^2)}{2 \times 2} = \frac{25 - 16}{4} = \frac{9}{4} = 2.25 \text{ ms}^{-2}$$

$$= 2\, ms^{-2}\ (to\ 1\ sig\ fig)$$

M2.4 Solve algebraic equations, including quadratics

Mathematical concepts

Learners should be able to:

- Solve linear equations
- Solve quadratic equations

The skills learners need to solve linear equations are similar to those used when rearranging a formula to make another variable the subject. Starting from the basic form

$$y = ax + b$$

This can be solved by first subtracting *b* from both sides

$$y - b = ax$$

then dividing both sides by *a*

$$\frac{y-b}{a} = x$$

and switching the LHS and RHS to change it to a more familiar representation

$$x = \frac{y-b}{a}$$

Quadratic Equations

The quadratic equation has the basic form

$$ax^2 + bx + c = 0$$

with $a \neq 0$. The number of solutions depends on the value of the discriminant

$$D = b^2 - 4ac$$

For D>0 there are two solutions

For D=0 there is one unique solution

For D<0 there are no *real* solutions.

In A Level Physics only real solutions are considered, however learners must be able to recognise when a quadratic equation has no real solutions.

The quadratic formula can be used to find the solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The formula will always work (provided there are real solutions). Other strategies for finding the solution are factorising and completing the square and learners may use them if they are familiar with the technique.

Contexts in Physics

- Solve linear equations in the context of suvat equations
- Determining static equilibrium
- Solve quadratic equations in the context of a suvat equation
- Use of $\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$ (restricted to thin converging lenses and real images)

Example

A car accelerates from $v = 2$ ms^{-1} at $t = 0$ s with a constant acceleration $a = 4$ ms^{-2}

$$v = u + at$$

Substituting values gives

$$5 = 2 + 4t$$

We subtract 2 from both sides

$$3 = 4t$$

then we divide both sides by 4

$$t = \frac{3}{4} = 0.75\ s$$

and we have found the solution.

Example

A particle is projected from ground-level vertically upwards with an initial velocity of 20 m s^{-1}. How long does it take to *first* reach 10 m?

Initially recognising the correct equation as $s = ut + \frac{1}{2}at^2$ and then substituting the values in

$$(a = -g = -9.8\ ms^{-2})$$

$$10 = 20t - 4.9t^2$$

This is a quadratic equation as the highest power of the unknown variable is 2. This is not yet in a suitable form to solve the equation using the quadratic formula. First we 'set to zero' collecting all terms on one side. Putting all the terms on the LHS

$$4.9t^2 - 20t + 10 = 0$$

First calculate the discriminant

$$D = b^2 - 4ac = 20^2 - 4 \times 4.9 \times 10 = 204$$

The discriminant is positive, there will be two real solutions for *t.*

The quadratic formula can then be applied

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

substituting in values yields

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4.9)(10)}}{2(4.9)}$$

The brackets around the negative numbers are necessary here to avoid any errors involving negative numbers. The solutions for *t* can then be calculated

$$t = \frac{20 \pm \sqrt{400 - 196}}{9.8}$$

$$t = \frac{20 \pm \sqrt{204}}{9.8}$$

$$t = \frac{20 \pm 14.3}{9.8}$$

There are two answers here

$$t = \frac{20 + 14.3}{9.8} = 3.5\ \mathrm{s}$$

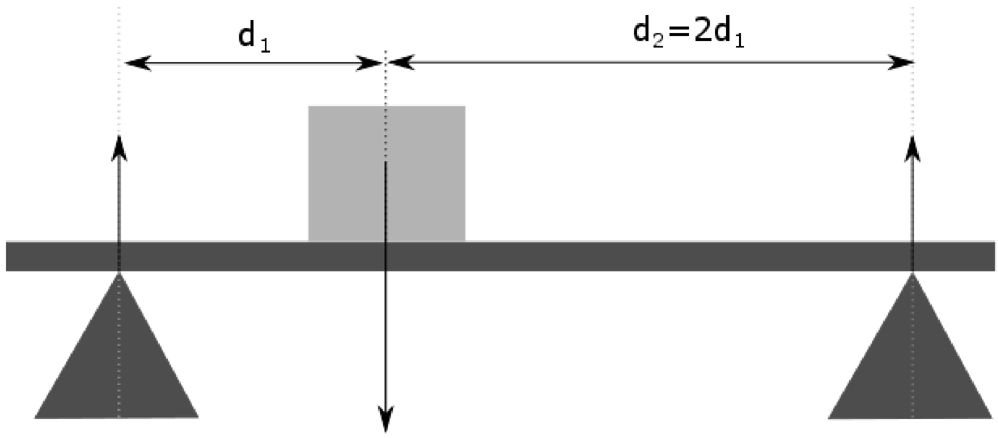
and

$$t = \frac{20 - 14.3}{9.8} = 0.58\ \mathrm{s}$$

Which is correct? Both are but the problem stated when the particle *first* reached 10 m and hence only *t*=0.58 is the correct solution to the problem. Notice all values have been taken to 2 sig fig as this was the level of precision g was quoted to.

Example

A block with a mass of 25 kg is at rest on a plank suspended on two pivots as shown below. Ignoring the mass of the plank, calculate the forces on the pivots.



To be at rest, the sum of all forces and the sum of all moments about any point must both be zero

$$F_1 + F_2 = w$$
$$M_1 = M_2$$

defining the moments around the centre of mass

$$F_1 d_1 = F_2 d_2$$

dividing both sides by d_1

$$F_1 = F_2 \frac{d_2}{d_1}$$

this expression can now be substituted into the force equation for F_1

$$F_2 \frac{d_2}{d_1} + F_2 = w$$

rearranging gives

$$F_2 = \frac{w}{1 + \frac{d_2}{d_1}}$$

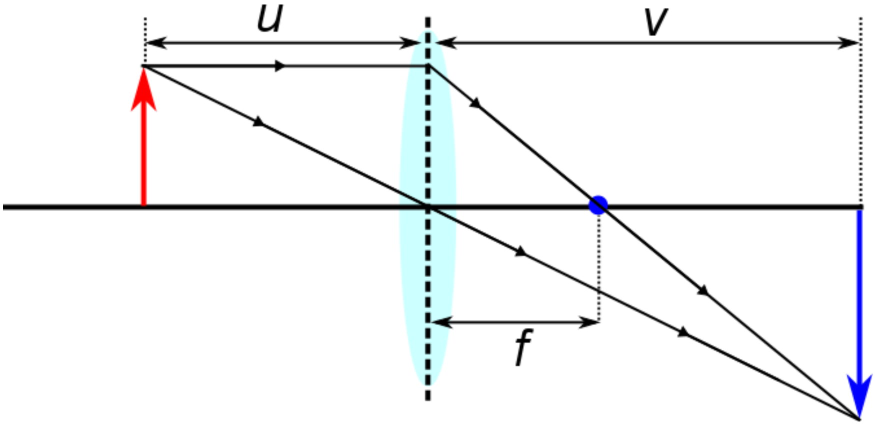
Filling in gives the result for F_2

$$F_2 = \frac{w}{1+2} = \frac{w}{3} = \frac{25 \times 9.81}{3} = 82\mathrm{N}$$

F_1can be found by substituting the result for F_2into the force equation

$$F_1 = w - F_2 = \frac{2}{3}w = 164\mathrm{N}$$

Example (Physics B only)



A converging lens is used to produce a magnified image, calculate the image distance v using the equation

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

The focal length $f = 30$ mm and the object distance $u = 100$ mm.

Filling in yields

$$\frac{1}{v} = -\frac{1}{100} + \frac{1}{30} = \frac{7}{300}$$

$$v = \frac{300}{7} = 43 \text{ mm}$$

M2.5 Use logarithms in context with quantities that range over several orders of magnitude

Mathematical concepts

Learners should be able to:

- perform calculations involving logarithms

Mathematical concepts

Logarithms are basically powers. If we take the following calculation

$10^2 = 100$

this can be expressed as

the power of 10 that gives 100 is 2

or in formal notation

$\log_{10} 100 = 2$

and we usually drop the 10 as it is assumed to be base 10 unless stated otherwise:

$\log 100 = 2$

Logarithms provide a better scale when dealing with quantities that vary exponentially (get big/small very quickly). For example, imagine sketching a graph where the scale goes from 10, 100, 1000, 10 000, 100 000 and so on. This would be impossible to do on a standard graph.

Taking the logarithms of these quantities gives 1, 2, 3, 4, 5, which is far more manageable to handle and to spot trends.

The natural logarithm is denoted by ln *x*, which is shorthand for $\log_e x$. Here e is a mathematical constant approximately equal to 2.7182818. This number is of central importance in mathematics, and often occurs in situations where quantities change exponentially over time. Like π, which learners should be aware of, it is an irrational number, meaning it cannot be represented as a repeating decimal or a fraction.

Contexts in physics

Example

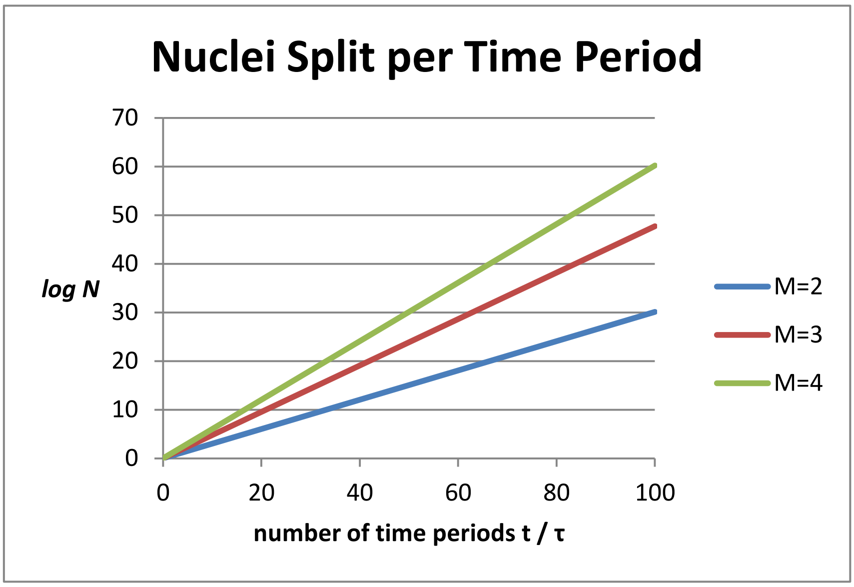
In a nuclear plant electrical energy is being generated through a process using the heat of fission from unstable nuclei. When a neutron with the right amount of kinetic energy collides with an unstable nucleus it splits into several smaller nuclei and emits a number of neutrons as well as other forms of radiation and heat. The emitted neutrons can split further atoms within the reactor.

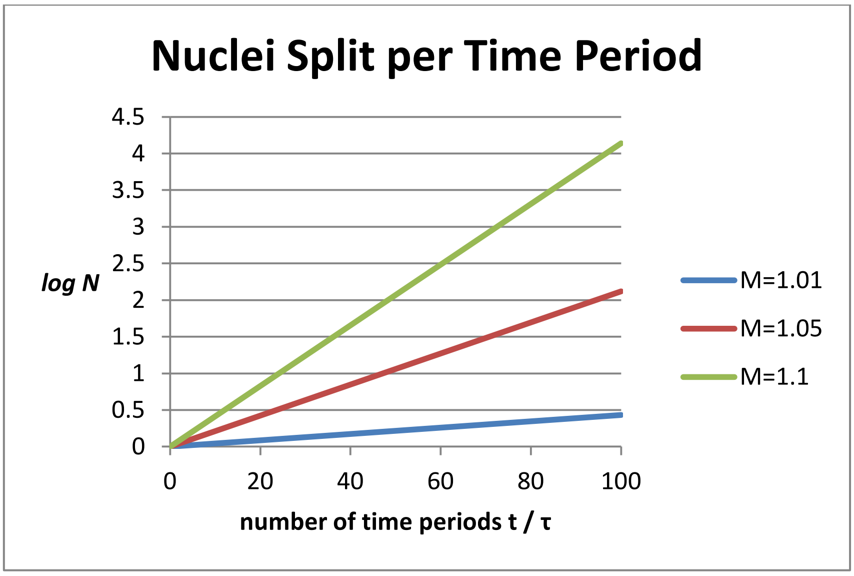
If the number of neutrons emitted is greater than 1, a chain reaction can occur and the number of atoms being split will increase exponentially. The thermal power of the nuclear reaction is proportional to the number of decaying nuclei. In a nuclear power plant, the multiplication factor needs to be very carefully controlled by absorbing excess neutrons, otherwise the results could quickly become disastrous.

In the model, we assume a simple model of the reaction, with a single nucleus decaying at t=0. The time each reaction takes is τ and the multiplication factor is M. The number of nuclei N decaying per unit time τ is then

$$N = M^{t/\tau}$$

If N is plotted on a linear scale, the graph will rise vary rapidly and it will be difficult to read. If log(*n)* is plotted against *t* the graph becomes much easier to read. The graphs below plot log(*N)* for several *M.* Even for relatively modest values of *M* a rapid increase of power of the reactor is possible.




Nuclei Split per Time Period
log N
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
20
40
60
80
100
number of time periods t / τ
M=1.01
M=1.05
M=1.1

M3 – Graphs

M3.1 Translate information between graphical, numerical and algebraic forms

Mathematical concepts

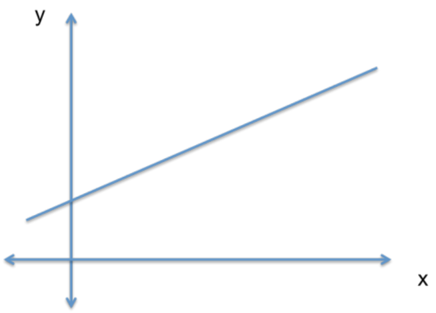
Learners should be able to:

- Interpret graphs of relationships between two variable to formulate formal algebraic/numerical formulae

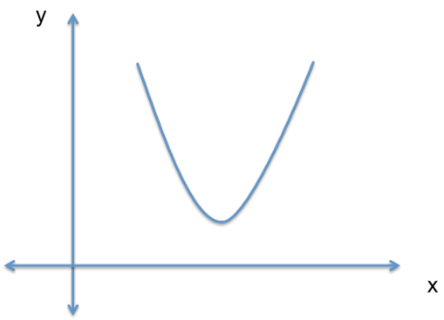
Learners will not be expected to undertake a full interpolation technique to obtain a relationship between variables (Newton's interpolation formula and Lagrange's method are two such advanced formulae). Learners must be able to use an appropriate algebraic form to make an estimate for the exact relationship.

Learners are expected to be able to identify the following set of general graphs

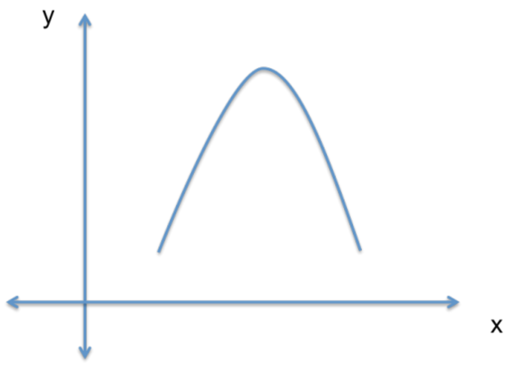
Linear $y = mx + c$



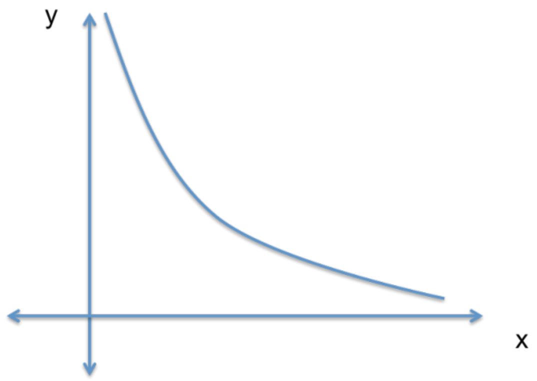
Quadratic (+) $y = ax^2 + bx + c$



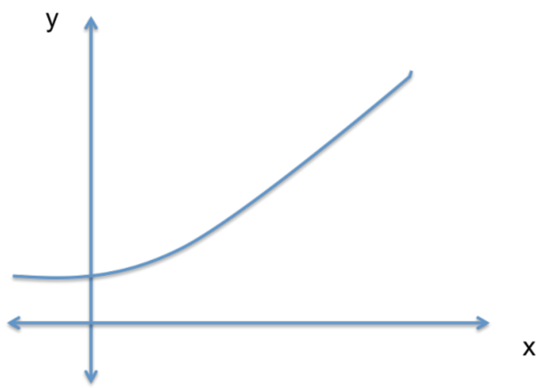
Quadratic (-) $y = -ax^2 + bx + c$



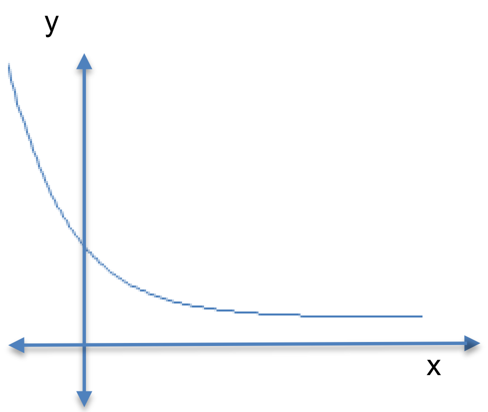
Reciprocal $y = \frac{k}{x}$ or an inverse square $y = \frac{k}{x^2}$ for x>0



Exponential Growth $y = Ae^{\lambda x}$



Exponential Decay $y = Ae^{-\lambda x}$



Contexts in Physics

Learners will be expected to:

- Interpret a graph and calculate key quantities and determine a relationship between two variables
- The graphical representation of the digitisation of an analogue signal for a given number of levels of resolution (Physics B only)

For example the Stress-strain curve will usually be illustrated graphically as a linear plot through the origin, hence the relationship

$$stress = E \times strain$$

here *E* is Young's Modulus. Other examples of linear relationships are Hooke's law and a velocity-time relationship under constant acceleration.

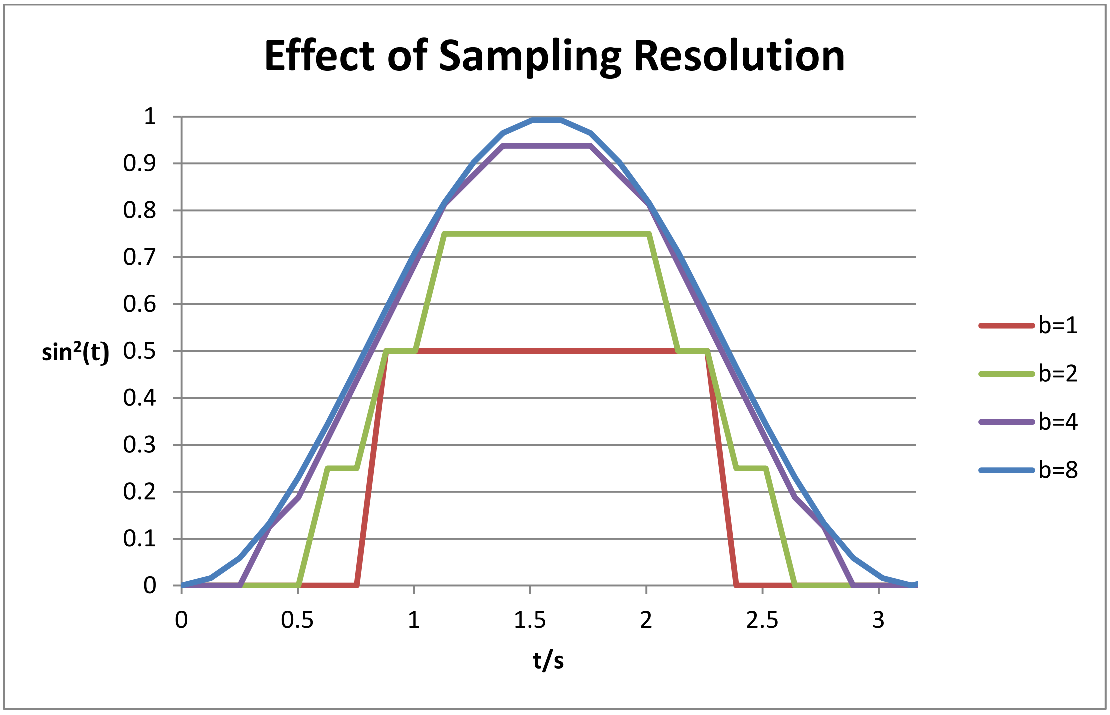
Example (Physics B only)

When an analogue signal is converted to digital, the correct sampling frequency and resolution must be chosen to ensure the signal is correctly captured. The number of alternatives *N* is

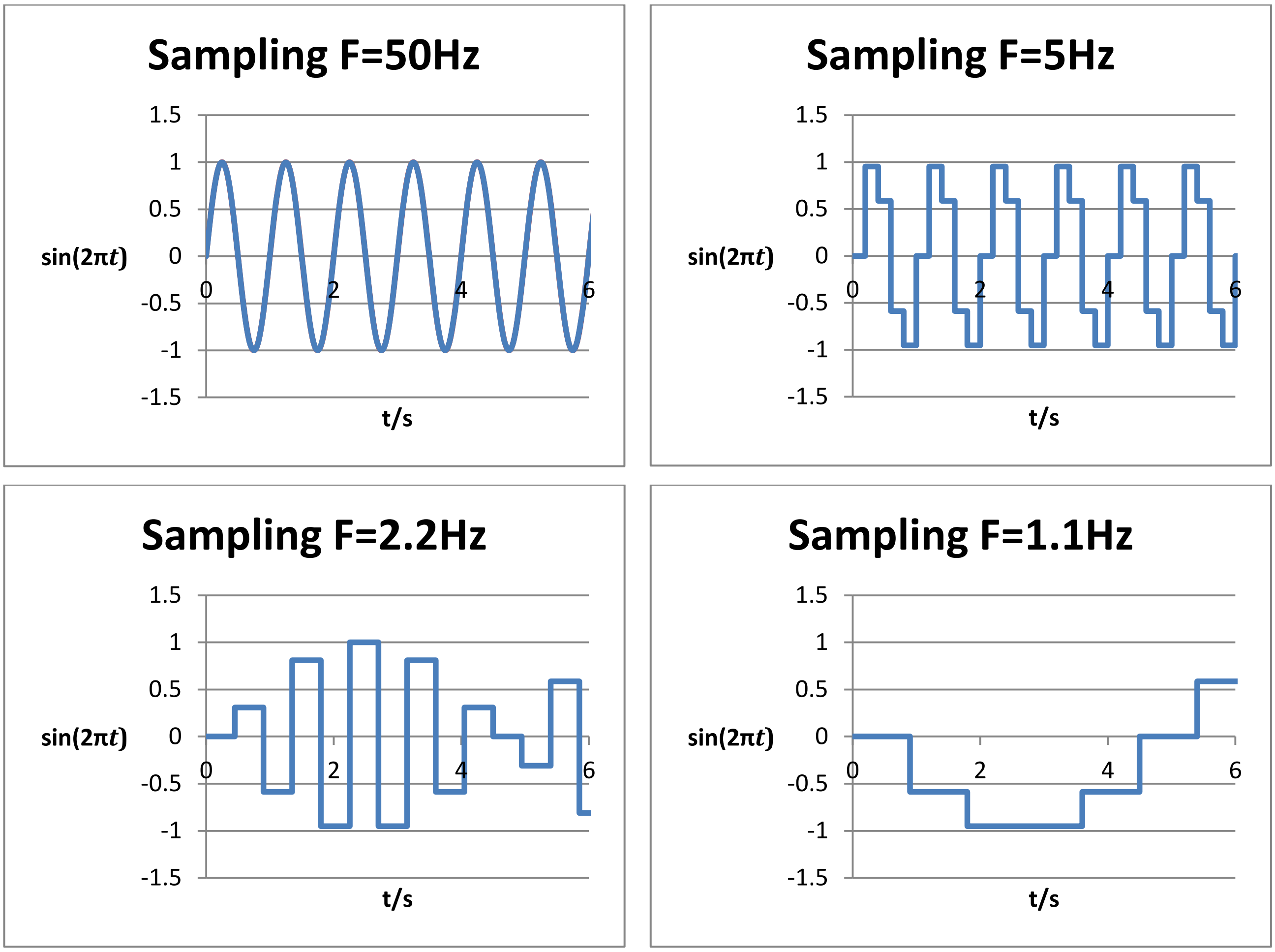
$$N = 2^b$$

with *b* the number of bits.

If *b* is too low, small variations in the signal are not captured. An example is shown for $y = \sin^2(t)$



The sampling frequency determines the highest frequency of the signal that can be measured. The minimum sampling rate required is twice the maximum frequency in the signal. A sine wave with $F = 1$ Hz is sampled at several frequencies and results are plotted below. Below the minimum sampling rate, at 1.1 Hz, the signal cannot be captured and a low frequency spurious frequency is introduced. At 2.2 Hz the signal can be recovered, although the quality of the signal can be increased by further increasing the sampling rate.



M3.2 Plot two variables from experimental or other data

Mathematical concepts

Learners should be able to:

- Plot a graph from experiment or other data on paper or in a spreadsheet

Plotting a graph on paper requires little mathematical knowledge but is more of an exercise in steady-hand movement and knowledge of the conventions. Rulers should be used and consideration of the scale of the graph should be taken into account *before* axes are drawn. Axes need labels in the form *Quantity*/Units and a title for the graph is essential.

Plotted points must be accurate to at least half a small square. Values from raw data will not always precisely represent the underlying relationship and learners will need to draw lines and curves of best fit. There will also be uncertainties in the measurements which can be represented by error bars. When drawing a line or curve of best fit, there must be a reasonable balance of points about the line. A ruler should be used for a line of best fit, and curves should be smooth.

If any of the variables do not start from 0 a zig-zig line should *not* be drawn, the scale simply starts at a sensible starting value. Care should be taken when taking intercepts in this case.

Contexts in physics

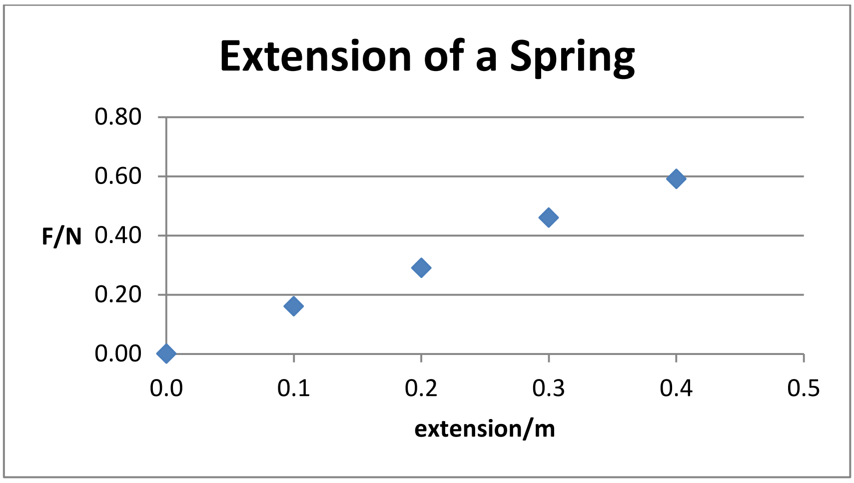
- Draw a graph demonstrating the relationship between two variables

Example

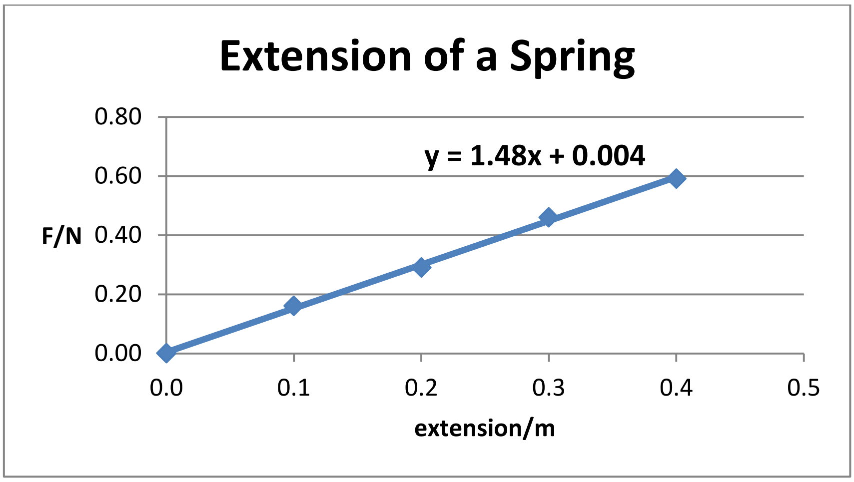
The extension from a spring was measured along with the force applied and put into a spreadsheet

	A	B
1	Extension x/m	Force/N
2	0.0	0.00
3	0.1	0.16
4	0.2	0.29
5	0.3	0.46
6	0.4	0.59

To find Hooke's constant a graph needs to be plotted, in Excel click on 'Charts' and choose a Scatter Graph. Select the data by highlighting the appropriate cells to get the graph.



To find Hooke's constant the gradient of the line needs to be calculated. In Excel the gradient can be found by right-clicking on the line and adding a "trendline". The equation can then be displayed on the chart. In this case y=1.48x+0.004. The value in front of *x* is the gradient and hence the Hooke's constant is k = 1.48 Nm^{-1}.



Notice the line is strictly only valid for the range of data values that is given. Beyond the given data points we cannot assume that the exhibited behaviour continues, great care must thus be taken when extrapolating data.

M3.3 Understand that y=mx+c represents a linear relationship

Mathematical concepts

Learners should be expected to know:

- A linear relationship can be written as $y = mx + c$
- m is the gradient of the line
- c is the *y* intercept of the line

Learners should be able to work out a relationship given a graph and learners should be able to sketch a graph given a linear relationship. The former is dealt with in the next sections but the latter is explained now.

Learners should understand that a positive *m* represents a line going 'up' from left to right and a negative m a line going 'down' from left to right. When sketching the graph from the equation they should always start from the *y*-intercept if practical and then use the equation to determine another point on the graph sufficiently far away from the y-axis. Once this extra point has been plotted the line can be drawn.

Contexts in Physics

- Sketch a graph showing a linear relationship from the formula linking the two variables

In the SUVAT equations the relationship between the velocity and time of an object travelling under uniform acceleration is given by:

$$v = u + at$$

This may not at first be considered as a linear graph but by re-ordering the terms on the right hand side to give

$$v = at + u$$

learners can recognise this as in the form *y* = *mx* + *c* but with *v* and *t* representing *y* and *x* respectively and a representing the gradient and *u* representing the initial velocity. Imagine that the particular relationship is

$$v = 0.8t + 3$$

To sketch the graph start from the *y* intercept (0,3). Then determine the *y* when *t=10*, this gives (10,11). After this point is plotted the graph can be plotted. Additional points can be added to the graph if required.

M3.4 Determine the slope and intercept of a linear graph

M3.5 Calculate rate of change from a linear graph

Mathematical concepts

Learners should be able to:

- Find the y-intercept of a linear graph
- Find the gradient of a linear graph
- Calculate rates of change from a linear graph

To find the y-intercept learners examine where the line crosses the y-axis. To find the gradient the following concept can be used

$$Gradient = \frac{'Rise'}{'Run'}$$

The 'Rise' represents the vertical step between 2 points and the 'Run' represents the horizontal step. The rise could be negative and care has to be taken in this case to respect the sign. The principle is that we take two points on the line and choose a 'starting point'. From this point, measure the horizontal distance to the other point ('run') and the vertical distance to the other point ('rise') and then perform the division to find the gradient.

If drawing a graph from measured values, in calculating the gradient it is advisable to draw a 'triangle' showing the rise/run on the graph and label the co-ordinates of the vertices. The width of any triangle used should be at least half the width of the graph.

The principle is that two points are taken on the line of the graph. Measuring the horizontal distance between the points gives the run, and the vertical distance gives the rise.

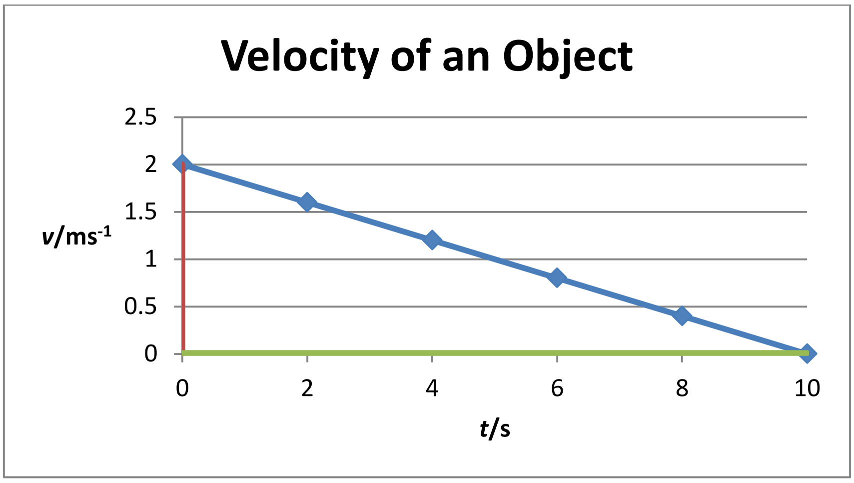
When determining the gradient of a graph plotted from experimental data, the points used must be on the line of best fit. Learners must **not** just use two of the plotted points to determine the gradient.

Contexts in Physics

- Find initial velocity and acceleration from a constant acceleration graph
- Interpret the gradient as a 'rate of change'

Example

Examine the following velocity –time graph representing an object moving under constant acceleration



The relationship between velocity and time is of the form

$$v = u + at$$

and hence the *y*-intercept is *u* and the acceleration is the gradient.

The *y*-intercept is (0,2) and thus the initial velocity is 2 ms^{-1}. To find the gradient and hence the acceleration choose another a point on the line. The obvious choice is the point on the x intercept (10,0).

The 'Rise' is -3 as to get from (0,2) to (10,0) requires a 'drop' of 2 and hence a rise of -2. The 'Run' is 10 as to get from (0,2) to (10,0) requires a 'run' of 10. The acceleration can be calculated as:

$$a = \frac{-2}{10} = -0.2\ \mathrm{ms}^{-2}$$

Example

For example Hooke's law states that $F = kx$. In a graph of *F* against *t*, *k* represents the gradient and is also the rate of change of *F* with respect to *x*. In words, how quickly *F* changes as *x* changes.

M3.6 Draw and use the slope of a tangent as a measure of a rate of change

Mathematical concepts

Learners should be able to:

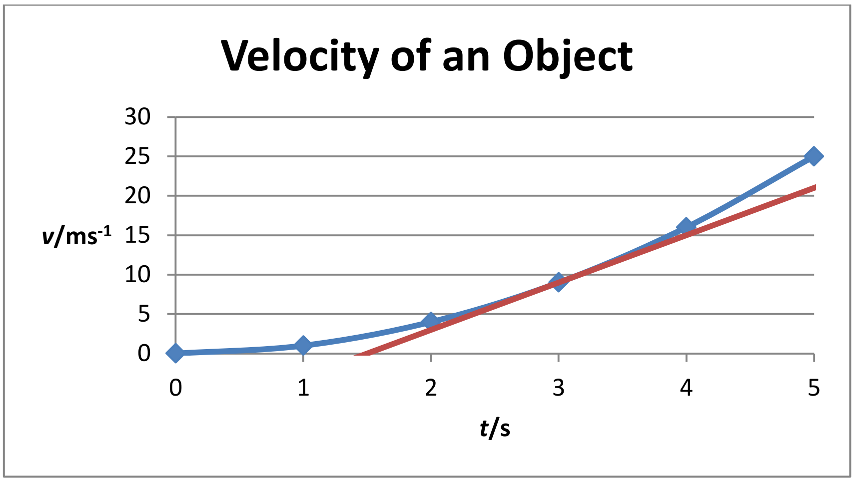
- Draw a tangent to a curve at a given point
- Find the gradient of the tangent

For linear graphs the gradient is the same throughout and hence the rate of change is easy to obtain. For non-linear a tangent has to be drawn to approximate the instantaneous rate of change. Non-linear graphs have an ever changing gradient and hence the rate of change will change from point to point.

Contexts in physics

- Understand the gradient of a velocity-time graph represents the rate of change of velocity with time and is the acceleration

Take for example the following graph representing a relationship between velocity and time. To find the instantaneous rate of change when *t* = 3 a tangent has to be drawn



To find the gradient two points are selected on the tangent and through the 'Rise'/'Run' calculation the instantaneous rate of change can be found. By taking two points on the tangent line, (1.5,0) and (4,15), the gradient, and instantaneous rate of change, at *t* = 3 can be determined

$$a = \frac{15 - 0}{4 - 1.5} = 6 \text{ m s}^{-2}$$

M3.7 Distinguish between instantaneous and average rate of change

Mathematical concepts

Learners should be able to:

- Understand the difference between instantaneous rate of change and average rate of change

Instantaneous rate of change represents the rate of change at a given moment or place in time or space.

The average rate of change is exactly that; an average rate of change taken over the global domain. For velocity it is given by

$$Average\ velocity = \frac{final\ displacement - initial\ displacment}{Change\ in\ time}$$

This could be vastly different from the instantaneous velocity and is representative of the whole motion rather that the rate of change at a given time.

Average velocities can be counter-intuitive. Imagine a racing car driving around a track of 10 km length. After completing 40 laps in 2 hours, the car crosses the start/finish line. The *average velocity* during the race would be zero, while at no point during the race the *instantaneous velocity* was zero.

Velocity and displacement are vectors, therefore they have a direction. Displacements in opposite directions do cancel each other out. The path the object followed between the initial and final position has no influence on the average velocity.

Speed is a scalar. It would therefore be incorrect to say that the racing car in the example above has an average *speed* of zero. The path followed is important here, the distance travelled is 400 km in 2 hours, giving an average speed of 200 km/h.

Contexts in physics

- Find the average acceleration of an object

As in the example of the previous example the instantaneous velocities at times *t*=0,1,2,3,4 are *v*=0,1,4,9,16 respectively. The average acceleration is:

$$Average\ acceleration = \frac{\Delta v}{\Delta t} = \frac{(16 - 0)}{4} = 4 \text{ m s}^{-2}$$

which is representative of the entire motion.

The instantaneous acceleration continuously changes, and cannot be found by taking a simple average. The method learners are expected to be familiar with is plotting a graph and drawing a tangent, as discussed in M3.6.

M3.8 Understand the possible physical significance of the area between the curve and the x axis and be able to calculate or estimate it by graphical methods as appropriate

Learners should be able to:

- Find the area underneath a curve by numerical methods
- Understand what the area underneath a curve represents

This is the other side of Calculus, Integration. Without a formal approach the following should be noted and learnt:

- Area underneath a velocity-time graph represents the *overall* displacement
- Area underneath a voltage-charge graph is energy stored

There are a number of points to be made about this. Firstly in the context of kinematics a velocity-time graph when v is negative and thus below the x axis this represents a negative *displacement (vector)* but a positive *distance travelled (scalar)*. Hence if displacement is sought the sign is to be respected but if a distance travelled is required the absolute value is taken.

Without a thorough understanding of The Fundamental Theorem of Calculus (Differentiation and Integration are inverses of each other) learners could be excused for questioning these concepts. Finding the gradient and then finding the area are inverses to each other mathematically. Hence finding the gradient of a displacement graph gives the velocity but going the other way, finding the area of a velocity graph gives us the displacement. This cannot be explained thoroughly without resorting to a formal mathematical argument but can be explained by the formula for velocity:

$$v = \frac{\Delta s}{\Delta t}$$

This is effectively a 'rise'/'run' calculation for the gradient of a displacement-time graph. Rearranging gives:

$$\Delta s = v \times \Delta t$$

Now the change in displacement is given as the product of velocity and change in time. This is effectively the formula for the area of a rectangle (height x length) with height velocity and length Δt and hence the area underneath the velocity-time graph.

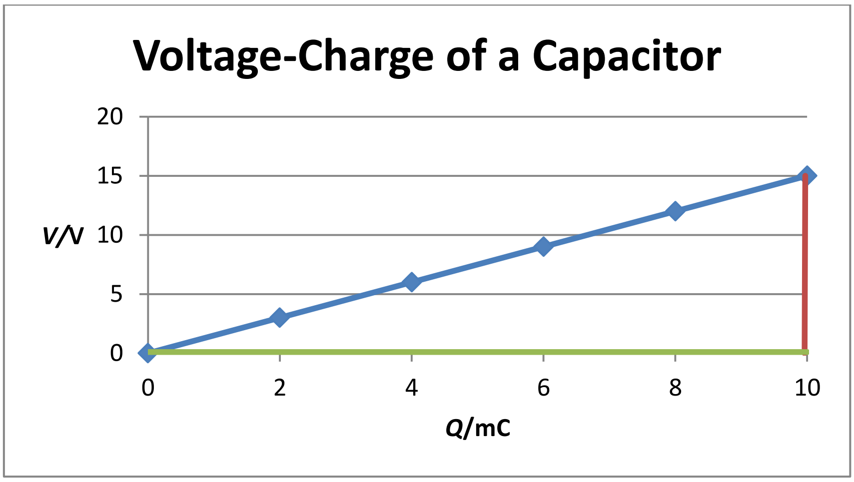
If the graph is a series of straight lines then using the formulae for areas of rectangles, triangles and trapeziums will give the exact area. Often this is not the case and an estimate has to be found by using counting rectangles as demonstrated the example below.

Contexts in physics

- Find the area underneath a voltage-charge graph to find the stored energy of a capacitor
- Impulse is equal to the area under a force-time graph.

Example

To determine the energy stored on a capacitor the following measurements were made



To find the energy stored the area under the curve has to be found. Since the graph is a straight line, a single trapezium can be drawn. The area of the trapezium is given by

$$Area = \frac{1}{2} \times (sum\ of\ parallel\ sides) \times (distance\ between\ parallel\ sides)$$

Choosing (0,0) and (10,15) as points and filling in

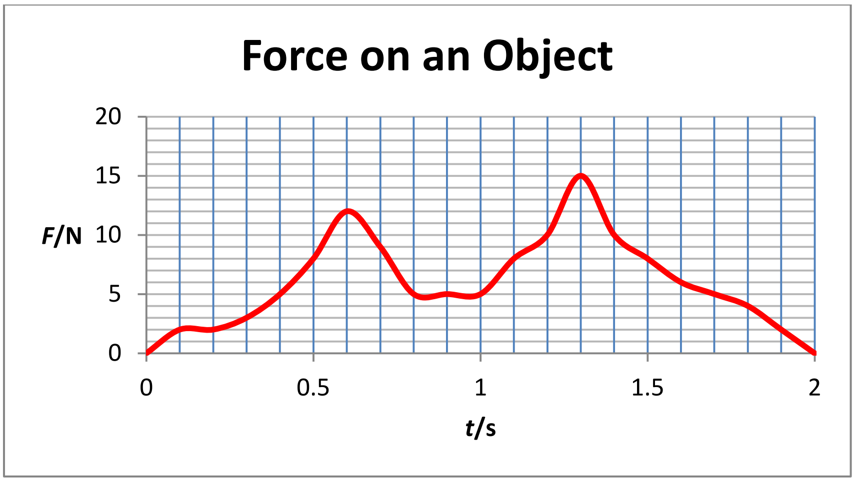
$$Area = \frac{1}{2} \times (0 + 15) \times 10 = 75$$

Appropriate units need to be read from the graph, note that charge is given in mC. The stored energy is thus 75 mJ or 7.5×10^{-2} J in standard notation.

Example

Another method to estimate the area involves counting rectangles, where the graph is plotted on a grid. By counting the number of rectangles the change in momentum can be determined. The area of each rectangle is $0.1 \times 1 = 0.1$ kg m s^{-1}. The change in momentum is then

$$\Delta s = n_{\text{rectangles}} \times 0.1 = 303 \times 0.1 = 30.3 \text{ kg m s}^{-1}$$



M3.9 Apply the contexts underlying calculus (but without using the explicit use of derivatives and integrals) by solving equations involving rates of change using a graphical or spreadsheet approach

Mathematical Concepts

- Solve pseudo differential equations using a graphical/spreadsheet approach

Solving differential equations formally is a high-level mathematical skill. However whenever a rate of change is involved an approximate solution can be found by using the graph.

The equation

$$\frac{\Delta x}{\Delta t} = -\lambda x$$

is an example of a pseudo differential equation where the unknown now is a function *x* to be found as a function of *t*. Remember that the quantity on the LHS represents a rate of change and hence a gradient on a graph of *x* against *t*. The instantaneous gradients can be calculated point by point and then compared with the values of *x* at these points to find the value of lambda. Understanding that this equation represents exponential decay learners can then write the relationship in the form:

$$x = Ae^{-\lambda t}$$

where *A* is to be found by looking at the initial value of *x*.

Contexts in Physics

The resource 'Exploring the exponential function in Physics' discusses the exponential function and how it can be modelled.

M3.10 Interpret logarithmic plots

M3.11 Use logarithmic plots to determine exponential and power law relationships

Mathematical Concepts

Learners should be able to:

- Determine relationships between variables using logarithms

From a set of data a mathematical relationship that links the two variables is to be found. The two models that are tested in this context are:

- $y = ka^x$
- $y = kx^n$

Notice in the first model the *x* variable is in the power and this is an example of an exponential function. The second model has the *x* variable as the base and this is a Power function.

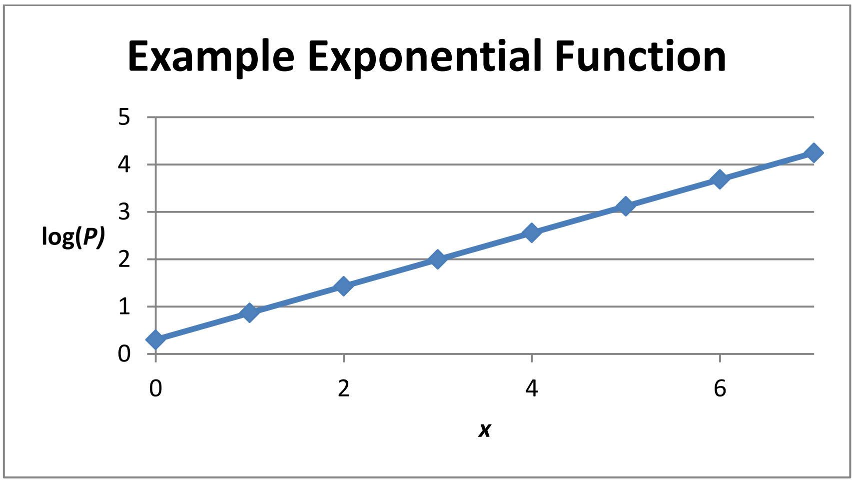
Applying these models to the set of data

x	0	1	2	3	4	5	6	7
P	2	7	27	98	359	1313	4807	17595

We will investigate the exponential model, ie $P = ka^x$. The aim is to find out the constants k and a. For the exponential model you should take the logarithm to base 10 of the DEPENDENT variable only; in this case P. Using a calculator and rounding to 2 decimal places gives

x	0	1	2	3	4	5	6	7
P	2	7	27	98	359	1313	4807	17595
log *P*	0.3	0.85	1.43	1.99	2.56	3.12	3.68	4.25

Now the graph of log *P* against *x* should be plotted. If the model is suitable then the graph will be a straight line



Learners are to draw a line of best fit and determine the gradient according to the method described in M3.4.

The y intercept is 0.3, whilst the gradient is $\frac{4.25-0.3}{7-0} = 0.56$. For the exponential model the value of *k* is given by $k = 10^{y\ intercept} = 10^{0.3} = 2$ whilst *a* is found by $a = 10^{gradient} = 10^{0.56} = 3.63$. Hence the model is written as:

$$P = 2 \times 3.63^t$$

Note that small rounding errors can lead to relatively large discrepancies between values for *P* even at modest values of *t*.

For a power relationship the method is roughly the same but for a few key differences. The differences are summed up in the table below:

Model	Logarithms	Graph	k	a
$P = ka^t$	Taking logarithms of *P* only	Plot log *P* against *t*	$k = 10^{y\ intercept}$	$a = 10^{gradient}$
$P = kt^a$	Take logarithms of *P* and *t*	Plot log *P* against log *t*	$k = 10^{y\ intercept}$	$a = gradient$

Contexts in Physics

- Obtain the time constant for capacitor discharge by interpreting the plot of log *V* by time

In this case the time constant is given by the gradient of a graph of log *V* against time

- Obtain the decay constant for radioactivity by interpreting the plot of log *N* by time

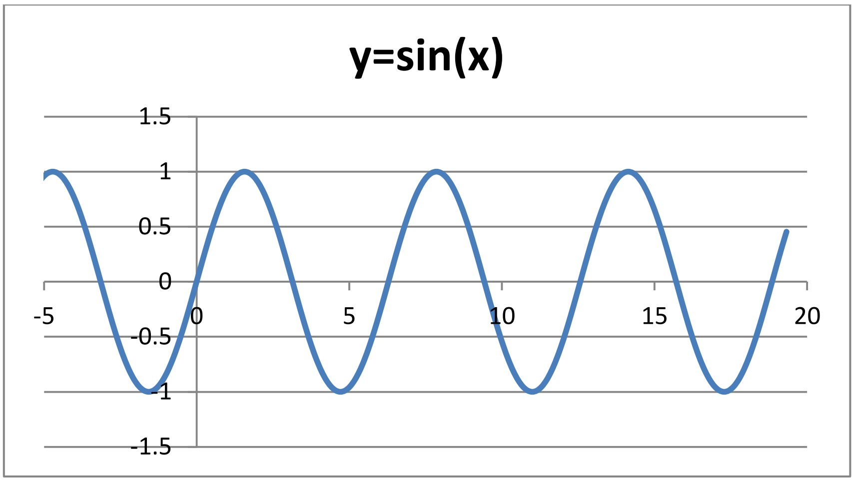
In this case the decay constant is given by the gradient of a graph of log *N* against time

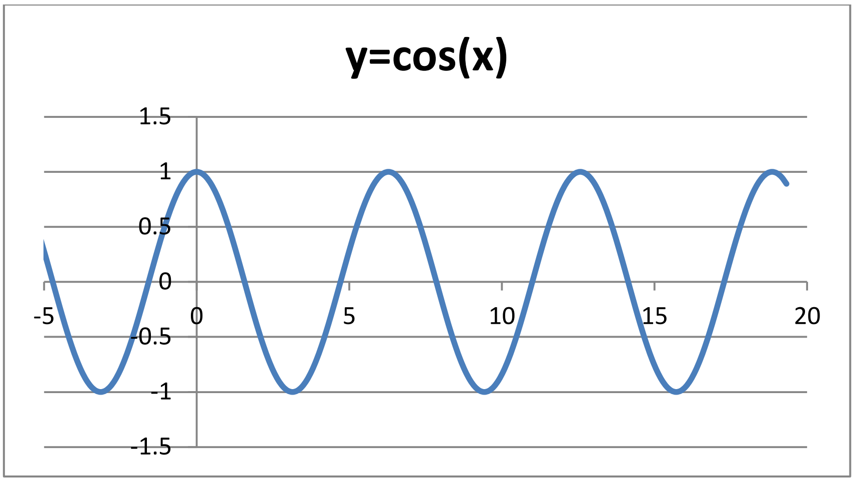
M3.12 Sketch relationships

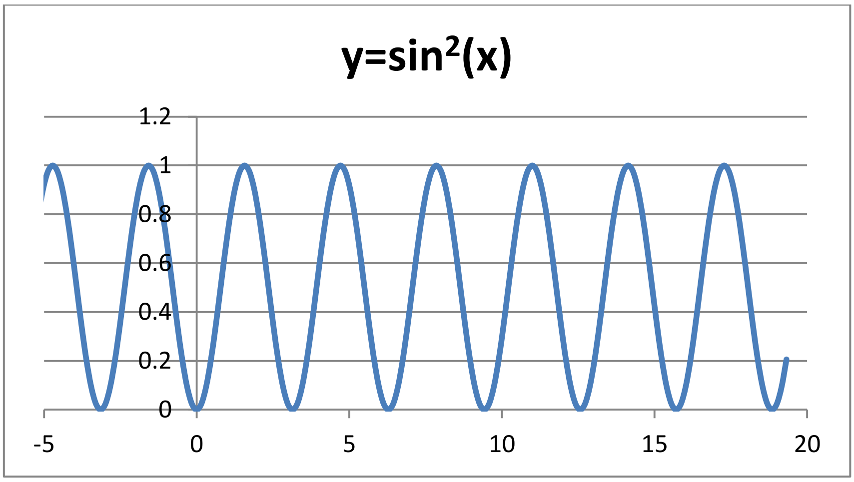
Mathematical Concepts

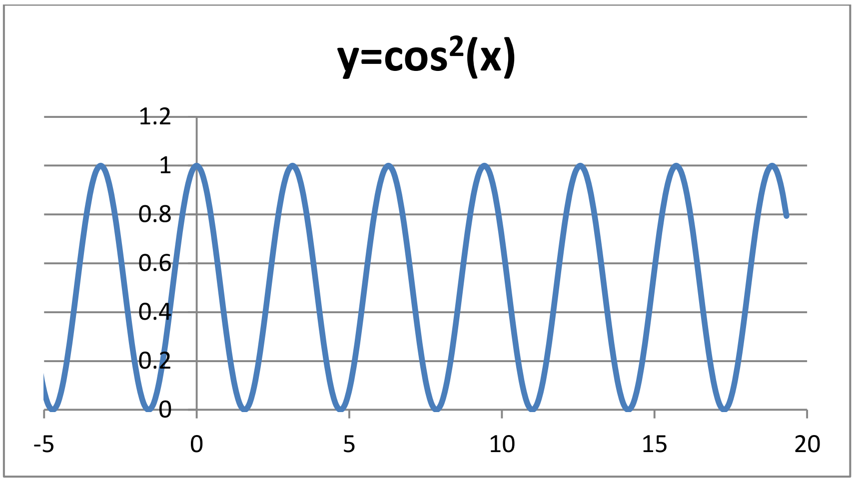
- Sketch graphs of different mathematical relationships

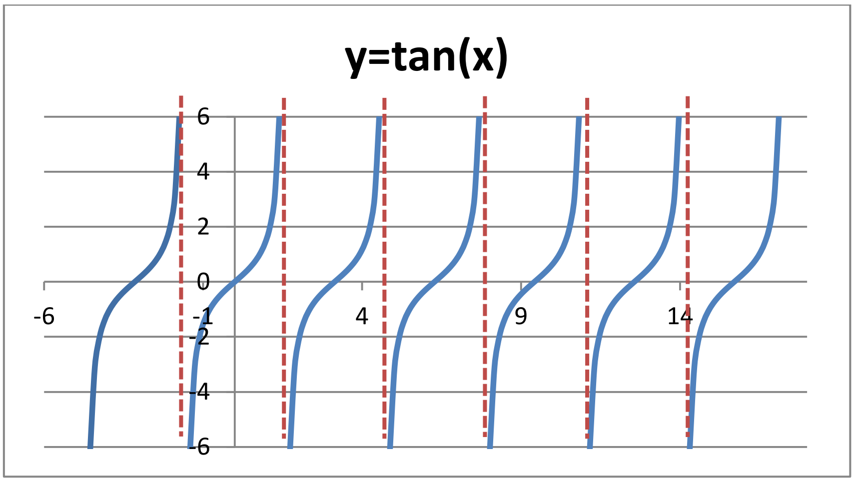
In addition to the graphs mentioned in M3.1 learners should recognise






y=sin²(x)
1.2
1
0.8
0.6
0.4
0.2
0
-5
0
5
10
15
20


y=cos²(x)
1.2
1
0.8
0.6
0.4
0.2
0
-5
0
5
10
15
20


y=tan(x)
6
4
2
0
-2
-4
-6
-6
-1
4
9
14

Contexts in Physics

- Sketch relationship between pressure and volume for an ideal gas
- Simple Harmonic Motion

Example

The ideal gas law states that:

$$pV = nRT$$

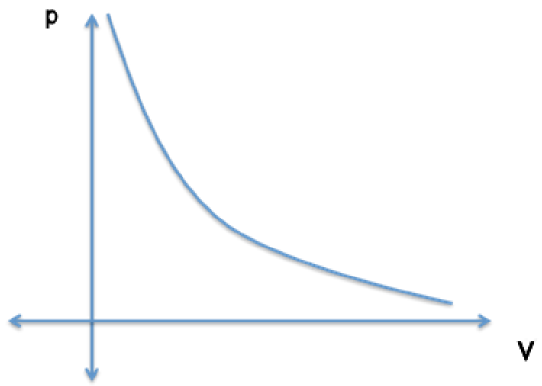
For a constant n, R and *T* then the relationship between *p* and *V* is given by:

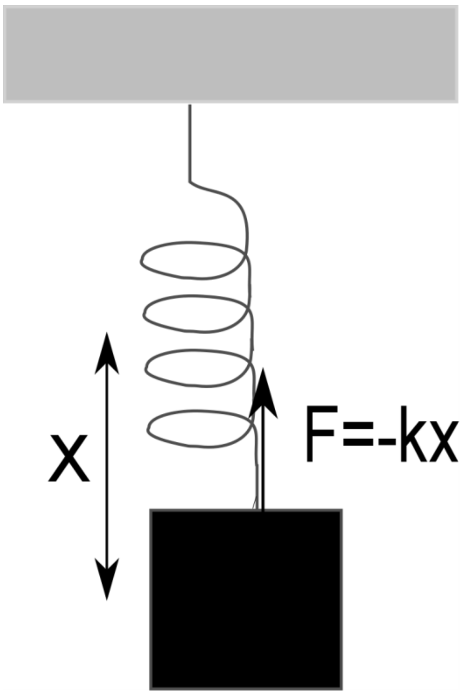
$$pV = constant$$

Rearranging this to make V the subject:

$$V = \frac{constant}{p}$$

This is an example of a reciprocal relationship with the general shape of the graph as shown.





Example

A mass attached to a spring undergoes Simple Harmonic Motion (SHM) when displaced from equilibrium as shown. For the purpose of the example, the effect of gravity is neglected. The force on the mass then equals

$$\mathrm{F} = -\mathrm{kx}$$

and the acceleration

$$\mathrm{a} = -\frac{\mathrm{k}}{\mathrm{m}}\mathrm{x}$$

The angular frequency of the SHM is denoted by ω and the defining equation of SHM is

$$\mathrm{a} = -\omega^2\mathrm{x}$$

The period *T* is related to ω by

$$\omega = \frac{2\pi}{\mathrm{T}}$$

By combining the two expressions for the acceleration it can be shown that

$$\omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$$

It is important to note that the angular frequency is independent of the amplitude.

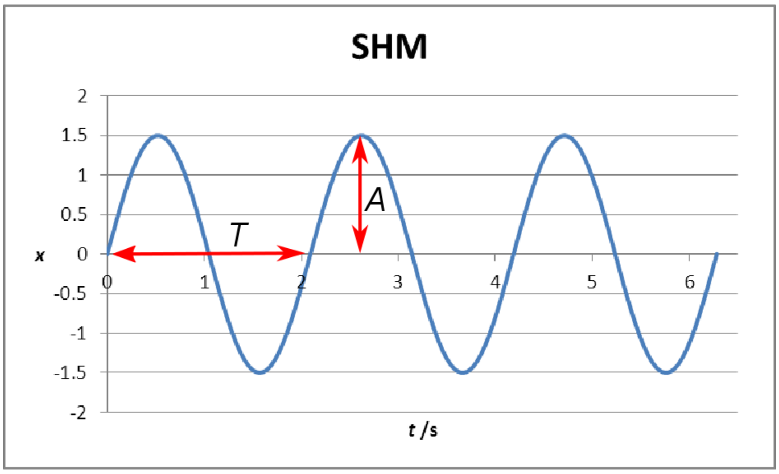
To find solutions for SHM requires solving a differential equation which is beyond A Level Physics. Learners are expected to know the resulting motion, which is sinusoidal

$$x = A \sin \omega t$$

or

$$x = A\cos \omega t$$

The only difference between the sin or cos form of the solution is the phase. At $t = 0$, $\sin \omega t = 0$ and $\cos \omega t = 1$, corresponding to a 90 degrees or $\pi/2$ phase difference. The amplitude *A* and period *T* can be read from the graph as shown below. In this case, *A*=1.5 and *T*=2.1



M4 – Geometry and Trigonometry

M4.1 Use angles in regular 2D and 3D structures

M4.2 Visualise and represent 2D and 3D forms including 2D representations of 3D objects

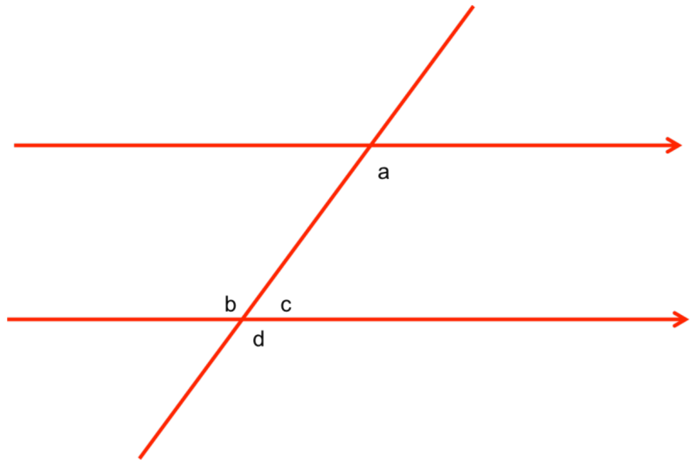
Mathematical concepts

Learners should be able to

- Know the sum of interior angles for a triangle and quadrilateral
- Apply knowledge of interior angle sums to find unknown angles

Learners are expected to know that the sum of interior angles of a triangle is 180° whilst in a quadrilateral it is 360°. In fact the formula for the sum of angles for an n-sided sided polygon is relatively simple at $180(n-2)$ although learners are not expected to memorise this relation in the context of the A level Physics course.

Angles round a point add up to 360°, whilst on a straight line add up to 180°. The parallel lines theorem is also applicable

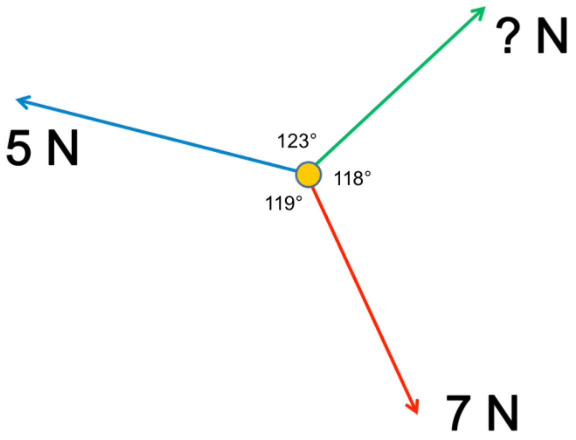


In this diagram a = d which are colloquially known as 'F' angles but formally known as *corresponding angles.* The angle a=b and these are known as 'Z' angles but formally known as *alternate angles*. Finally a+c=180 and are known as 'C' angles or *supplementary angles.*

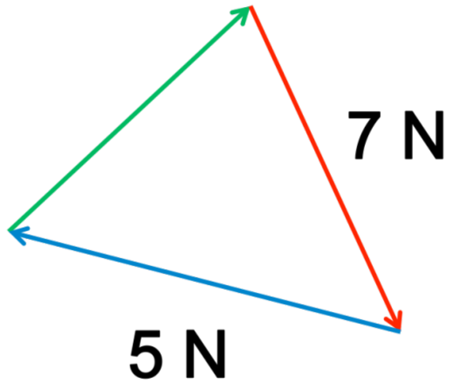
Contexts in Physics

- Find unknowns in force diagrams

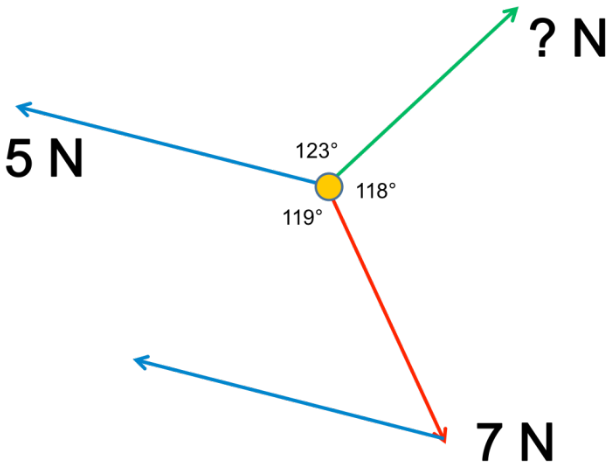
In the following force diagram the particle is in equilibrium, the force marked with a question mark needs to be determined



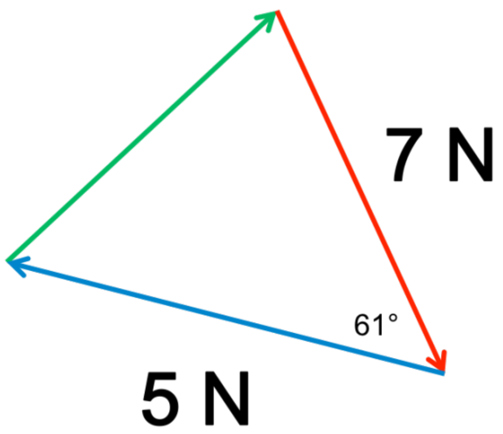
These are re-arranged to make a force triangle below as the particle is in equilibrium



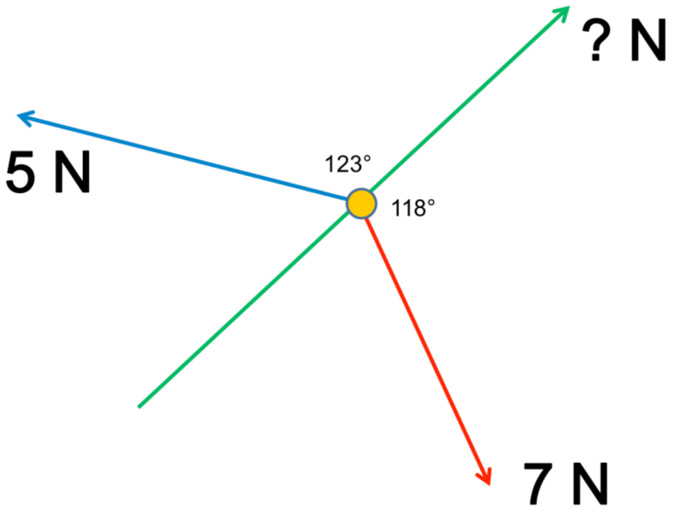
The missing force can be calculated but what are the angles. Knowledge of parallel line geometry and some geometrical reasoning will help here. From the first diagram imagine the blue vector 'sliding' down to hit the bottom of the red vector like so



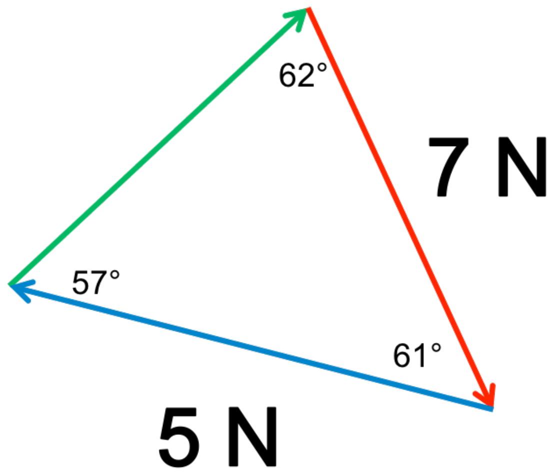
A supplementary angle can be calculated by finding 180-119=61 and hence one angle has been found



To find the angle at the top, imagine the green vector extending from the bottom like so



The angle at the top can be found by noticing that these are angles on a straight line and hence it is 180-118=62 and hence we have two angles and of course the third angle is found by subtracting both from 180: 180-61-62=57 and the angles in the diagram have been found



Now the forces can be found by applying the sin/cos rule (see later section)

M4.3 Calculate areas of triangles, circumferences and areas of circles, surface areas and volumes of rectangular blocks, cylinders and spheres

Mathematical concepts

- Find lengths and areas of triangles and circles
- Find surface areas and volumes of cuboids, cylinders and spheres

Learners are expected to be able to use the following list of formulae:

- Area of a right-angles triangle $\frac{1}{2}base \times height = \frac{1}{2}bh$
- Area of a triangle $\frac{1}{2}ab\sin C$, a and b are sides containing the interior angle C
- Circumference of a circle $2\pi r$ with r the radius
- Area of a circle πr^2
- Surface area of a cuboid $2(bh + bl + hl), b$ is base, l is length and h is height
- Volume of a cuboid hbl
- Surface area of a cylinder$2\pi r(r + l), r$ is the radius of the ends, l is the length of the cylinder
- Volume of a cylinder $\pi r^2 l$
- Surface area of a sphere $4\pi r^2$,
- Volume of a sphere $\frac{4}{3}\pi r^3$

Contexts in Physics

- Calculate the area of the cross-section to work out the resistance of a conductor given its length and resistivity

The formula

$$R = \frac{\rho l}{A}$$

gives the resistance of a wire R where ρ is the resistivity, l is the length and A is the cross-sectional area of the wire. For the wire $\rho = 1.7 \times 10^{-8}$ Ωm and $l = 1.00$ m. The cross-sectional area A has to be found in order to find R.

A wire with radius 1.20 mm is used. To find the cross-sectional area use the formula for area of a circle

$$\pi r^2 = \pi \times 0.00120^2 = 4.52 \times 10^{-6}\text{ m}^2$$

and substituting in the other variables, R can be found

$$R = \frac{\rho l}{A} = 3.8 \times 10^{-3}\Omega$$

M4.4 Use Pythagoras's Theorem and the angle sum of a triangle

Mathematical concepts

Learners should be able to:

- Apply Pythagoras's theorem in problems involving right-angled triangles
- Know the angle sum for a triangle and apply to problems involving triangles

Applying the angle sum of a triangle has been covered in M4.1. Pythagoras's theorem is stated as

$$a^2 + b^2 = c^2$$

where a, b are the two shorter sides of a right-angled triangle and c is the *hypotenuse* of the triangle.

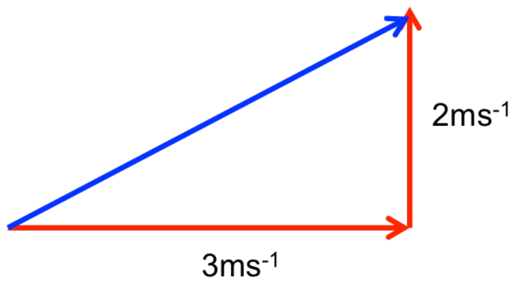
Contexts in Physics

- Find speeds from vectors representing velocity

Imagine an object being projected from the origin with initial velocity (correct to 2 significant figures)

$$u = \begin{pmatrix} 3.0 \\ 2.0 \end{pmatrix}$$

This can be represented by a triangle:



The speed can be calculated by applying Pythagoras's theorem to this triangle to find the hypotenuse

$$3^2 + 2^2 = speed^2$$

$$13 = speed^2$$

$$speed = \sqrt{13} = 3.6\ ms^{-1}$$

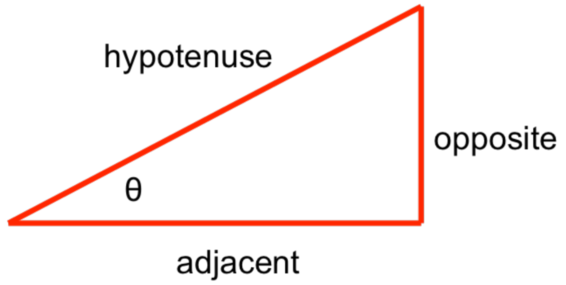
This method can also be applied to find the magnitude of a force when given in vector form.

M4.5 Use sin, cos and tan in physical problems

Mathematical concepts

- Know and apply the trigonometrical ratios in relation to right-angled triangles
- Know and apply the sine and cosine rule for non-right angled triangles

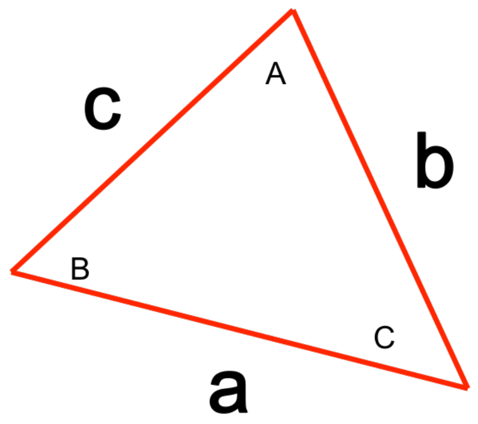
For a right-angled triangle



the following ratio's apply

$$\sin\theta = \frac{o}{h}$$
$$\cos\theta = \frac{a}{h}$$
$$\tan\theta = \frac{o}{a}$$

whilst for a non right-angled triangle



the rules are

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
$$c^2 = a^2 + b^2 - 2bc\cos C$$

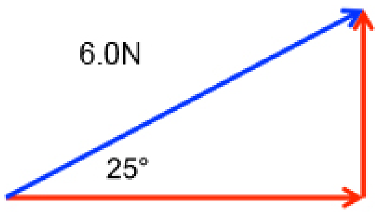
Notice that in the non-right angled triangles the sides and angles are labelled according to the side they are opposite to, with angles in capitals and sides in lower-case. So for example Angle A is opposite length *a*.

For the right-angled triangles the opposite and adjacent are labelled corresponding to whatever angle is given in the triangle (or the angle that is to be found).

Contexts in physics

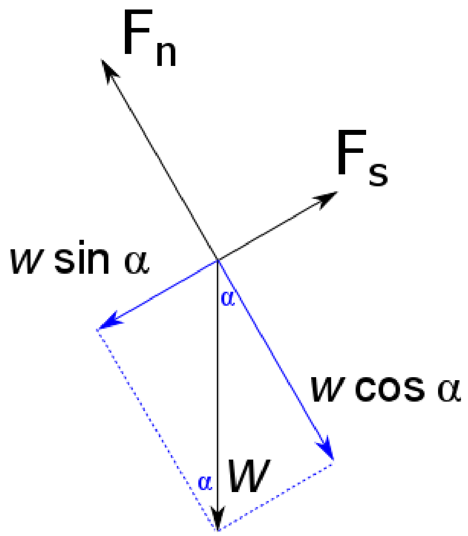
- Resolve forces into components

Take a force which has magnitude 6.0 N and has a direction of $25°$ to the horizontal as below



To find the vertical and horizontal components trigonometry has to be used.

To find the horizontal component, notice that this is 'next' to the angle given and represents the 'adjacent' side. The hypotenuse is the magnitude of the force and we now have a and h. The trigonometric ratio which contains these variables is the $\cos$ ratio and hence



$$\cos\theta = \frac{a}{h}$$

$$\cos 25 = \frac{a}{6}$$

$$F_{horizontal} = a = 6.0 \times \cos 25 = 5.4\ N$$

Similarly for the vertical component which is identified as the opposite side to the given angle the $\sin$ ratio has to be used:

$$\sin\theta = \frac{o}{h}$$

$$\sin 25 = \frac{o}{6}$$

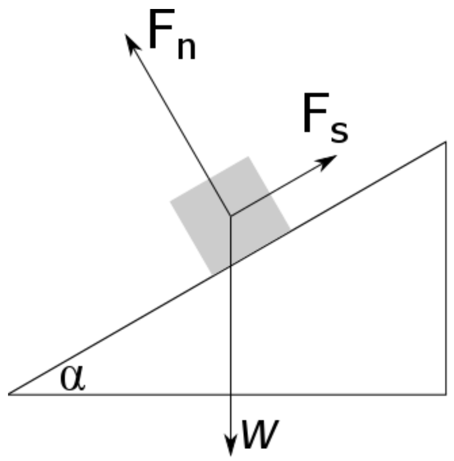
$$F_{vertical} = o = 6.0 \times \sin 25 = 2.5\ N$$

and the force has been resolved.

With enough practise these can be done very quickly but when a learner starts working on these problems a slow, measured approach is preferable. Once the horizontal component is found, the vertical component can be calculated using Pythagoras's theorem but when they become competent this is slower than using trigonometry. Take care that the calculator is in degree mode!

Example

A block of wood with m=1.0 kg is at rest on an incline with $\alpha = 30°$. The three forces acting on the block, gravity w, friction F_s and the normal reaction force F_n, are shown in the figure below. Calculate the magnitude of all the forces.



As the object is at rest, the sum of all forces must be zero. To find the magnitude of $F_n$and F_s we need to find the components of w parallel and perpendicular to the inclined plane. A force diagram can then be drawn.

By looking at the triangles in the drawing we can determine the components using simple geometry.

In the left triangle, w is the hypotenuse and the component parallel to the ramp is the "opposite" side. The parallel component is thus $w\sin(\alpha)$.

In the right triangle, w is the hypotenuse and the component perpendicular to the ramp is the "adjacent" side. The parallel component is thus $w\cos(\alpha)$.

The normal reaction and friction force are equal and opposite to the components of w and the result can now be calculated

$$F_s = w\sin(\alpha) = 1 \times 9.81 \times \sin(\alpha) = 4.9\ \mathrm{N}$$

$$F_n = w\cos(\alpha) = 1 \times 9.81 \times \cos(\alpha) = 8.5\ \mathrm{N}$$

A useful way for learners to remember if whether to use sin or cos to determine a component of a force is by looking at an "inclined" plane with an angle of zero. In this case it is obvious that $F_n = w$ and $F_s = 0$. As $\sin(0°) = 0$ and $\cos(0°) = 1$. It is then clear that to find F_n cos must be used and for F_s sin.

M4.6 Use of small angle approximations for sin, cos and tan

Mathematical concepts

Learners should know that for small values of θ:

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1$
- $\tan\theta \approx \theta$

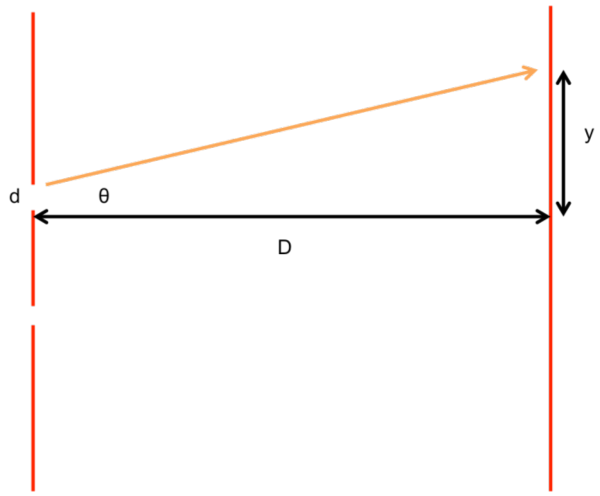
Formally these are the Taylor series approximation for the functions up to the linear term and should be treated with caution mathematically but in certain contexts of A level physics they are useful. Whenever the angle is in *radians* and is regarded as *small* these approximations apply. Usually this is applied in the context of Young's slit experiment.

Contexts in physics

- Calculate fringe separations in interference patterns
- Distance measured in astronomical unit (AU), light-year (ly) and parsec (pc)

Example

In the two-slit experiment waves travel through the slits at a small angle θ and cause an interference pattern on a screen a distance d away:



the length y can be calculated using the tan ratio:

$$\tan\theta = \frac{y}{D}$$

which using the small angle approximation is:

$$\theta \approx \frac{y}{D}$$

for there to be an interference maximum there has to be:

$$d\sin\theta = n\lambda$$

where λ is the wavelength of the beam and n is an integer. Using the small angle approximation gives:

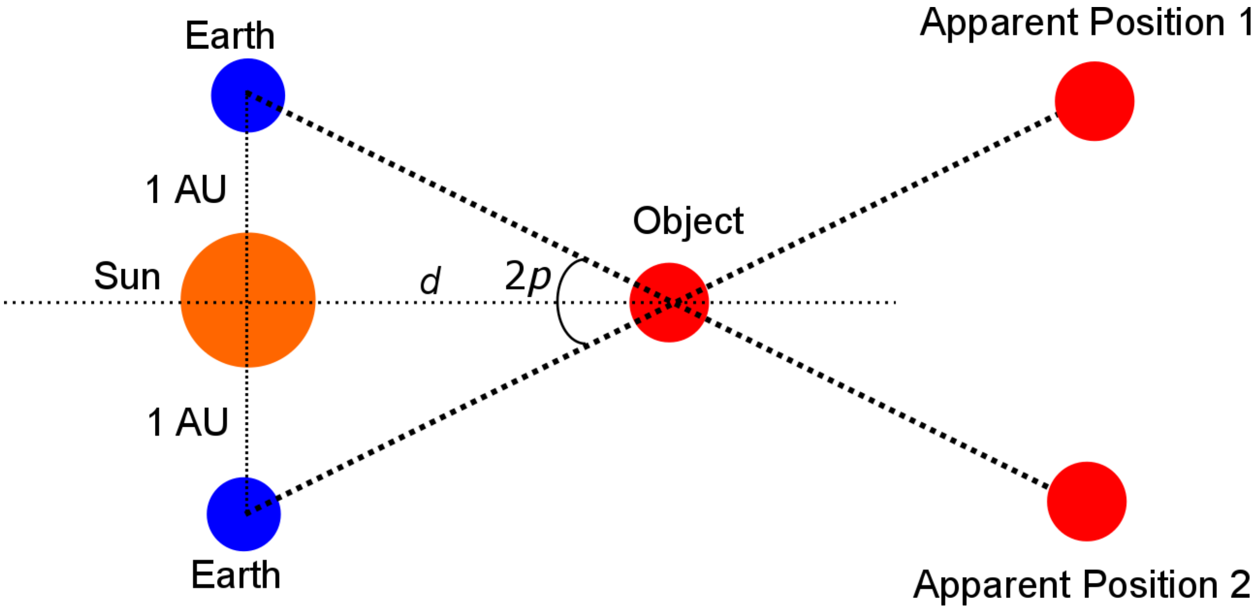
$$\theta = \frac{n\lambda}{d}$$

and substituting this into the first expression yields:

$$y \approx \frac{n\lambda D}{d}$$

Example

Early observations of the universe were made using triangulation. By measuring the apparent position of the same celestial body twice 6 months apart, the distance to the body can be determined from the parallax effect.



The average distance between the Earth and the Sun is defined as 1 Astronomical Unit (AU). Using the parallax effect a convenient measure for the distance to the celestial object is the parsec (from **par**allax arc **sec**ond). An arc second is equal to 1/60 arc minutes or 1/3600 degrees.

To find how much 1 parsec equals in AU, trigonometry can be used

$$\mathrm{d} = \frac{1}{\tan p}$$

As *p* is small we the approximation for small angles is valid

$$d = \frac{1}{p}$$

Filling in 1 arc second for *p*, converting from degrees to radians (see M4.7) finds the distance *d* which is rounded here to 3 significant figures

$$d = \frac{1}{(1/3600)\times(\pi/180)} = 2.06 \times 10^5 \text{ AU}$$

M4.7 Understand the relationship between degrees and radians and translate from one to another

Mathematical concepts

Learners should be able to:

- Convert between radians and degrees

As with any conversion problem the key mathematical concept here is of proportional reasoning. Actually in this concept it is very easy, all learners need to know and remember is that:

$$\pi \, radians = 180°$$

From this any conversion is possible. For example 30 degrees is 1/6 of 180 hence:

$$30° = \frac{\pi}{6}$$

Again $\frac{3\pi}{2}$ radians is 1.5 "lots" of π radians and hence

$$\frac{3\pi}{2} \, radians = 270°$$

If the radians are not given in a rational multiple of π then the following conversions are useful:

- Radians to degrees – times by 180 then divide by π
- Degrees to radians – times by π then divide by 180

Contexts in physics

- In circular motion, the angular speed ω of an object is the angle θ it moves through measured in radians (rad) divided by the time t taken to move through that angle. The unit for angular speed is the radian per second (rad s^{-1}).

  $$\omega = v/r = 2\pi f$$

  Where v is the linear speed in m s^{-1}

  r is the radius of the circle in m

  f is the frequency of the rotation in Hz

OCR customer contact centre

General qualifications
Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk

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